CERTAIN RESULTS ON THREE-DIMENSIONAL f-KENMOTSU MANIFOLDS WITH CONFORMAL RICCI SOLITONS

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ABSTRACT. In the present paper, we have studied conformal Ricci solitons on f-Kenmotsu manifolds of dimension three. Also we have studied ϕ -Ricci symmetry, η -parallel Ricci tensor, cyclic parallel Ricci tensor and second order parallel tensor in f-Kenmotsu manifolds of dimension three admitting conformal Ricci solitons. Finally, we give an example.

1. Introduction

In 1972, the notion of Kenmotsu manifolds was introduced by K. Kenmotsu in the paper [16]. f-Kenmotsu manifold is the generalization of Kenmotsu manifold. f-Kenmotsu manifolds has been studied by several authors such as Hui,Yadav and Patra [15], Venkatesha and Divyashree [23], Yildiz, De and Turan [24]. De and Sarkar introduced the notion of ϕ -Ricci symmetric manifolds in the paper [6]. η -parallel Ricci tensor and cyclic parallel Ricci tensor on three-dimensional quasi-Sasakian manifolds have been studied by De and Sarkar [5]. In 1926, Levy introduced the notion of second order parallel tensors. Later many author such as R. Sharma [21], [22], Chandra, Hui and Shaikh [4], Mondal and De [17] have studied second order parallel tensors on several manifolds.

The notion of Ricci solitons was introduced by R. S. Hamilton [14] which is the generalization of the Einstein metrics and is defined by

$$(\mathcal{L}_X g)(Y, Z) + 2S(Y, Z) + 2\lambda g(Y, Z) = 0,$$

where \mathcal{L}_X denotes the Lie-derivative along the vector field X, λ is a constant, S the Ricci tensor of type (0,2) and Y, Z are arbitrary vector fields on the manifold. Here X is called the potential vector field. A Ricci soliton is called shrinking, steady or expanding according as λ is negative, zero or positive. A Ricci soliton is the limit of the solutions of Ricci flow equation given by

$$\frac{\partial g}{\partial t} = -2S.$$

Many authors have studied Ricci solitons on different kind of manifolds. For details see ([3], [7], [8], [18]).

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Conformal Ricci flow equation was introduced by A. E. Fisher [13] in the year 2005 which is a variation of the classical Ricci flow equation and the equation is given by

$$\frac{\partial g}{\partial t} + 2(S + \frac{g}{n}) = -pg$$

and r = -1, where p is a time dependent non-dynamical scalar field, r is the scalar curvature of the manifold and n is the dimension of the manifold.

In 2015, the notion of conformal Ricci soliton was introduced by N. Basu and A. Bhattacharyya [2] which is the generalization of the Ricci soliton and the equation is given by

$$\mathcal{L}_X g + 2S = \left[2\lambda - \left(p + \frac{2}{n}\right)\right]g.$$

Conformal Ricci solitons have been studied in the paper [9] [10], [11].

In this paper we would like to study some properties of conformal Ricci soliton on three-dimensional f-Kenmotsu manifolds.

The paper is organized as follows: After introduction, we give some preliminaries in the Section 2. In Section 3, we have studied the conformal Ricci soliton on f-Kenmotsu manifolds. In Section 4, we heve derived some conditions of ϕ -Ricci symmetric f-Kenmotsu manifold admitting conformal Ricci soliton. Section 5 is devoted to study η -parallel Ricci tensor of an f-Kenmotsu manifold admitting conformal Ricci soliton. In Section 6, we have deduced some results of cyclic parallel Ricci tensor of an f-Kenmotsu manifold admitting conformal Ricci soliton. In Section 7, we heve studied second order parallel tensor and conformal Ricci solitons. In the last Section, we give an example.

2. Preliminaries

Let M be a (2n+1)-dimensional smooth manifold endowed with an almost contact metric structure (ϕ, ξ, η, g) , where ϕ is a tensor field of type (1, 1), ξ is a vector field, η is a 1-form and g is the Riemannian metric on M such that [15]

$$\phi^{2}(X) = -X + \eta(X)\xi, \quad \eta(\xi) = 1.$$

As a consequence, we get the following:

$$\phi \xi = 0, \quad g(X, \xi) = \eta(X), \quad \eta(\phi X) = 0,$$
$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y),$$
$$g(\phi X, Y) = -g(X, \phi Y), \quad g(\phi X, X) = 0,$$
$$(\nabla_X \eta)(Y) = g(\nabla_X \xi, Y),$$

for all vector fields $X, Y \in \chi(M)$. A differentiable manifold M of dimension (2n+1) with almost contact metric structure is said to be an almost contact metric manifold. M is said to be an f-Kenmotsu manifold if the covariant differentiation of ϕ satisfies

(1)
$$(\nabla_X \phi)(Y) = f\{g(\phi X, Y)\xi - \eta(Y)\phi X\},\$$

where $f \in C^{\infty}(M)$ is such that $df \wedge \eta = 0$. If $f = \beta$ is non-zero constant, then the manifold is called a β -Kenmotsu manifold. An f-Kenmotsu manifold is called

Kenmotsu manifold if f = 1. If f = 0, then the manifold is cosymplectic. An f-Kenmotsu manifold is said to be regular if $f^2 + f' \neq 0$, where $f' = \xi f$. For an f-Kenmotsu manifold, we get from (1)

(2)
$$\nabla_X \xi = f\{X - \eta(X)\xi\}.$$

The condition $df \wedge \eta = 0$ holds if $dim(M) \ge 5$ and does not hold if dim(M) = 3. In a three-dimensional f-Kenmotsu manifold, we have

$$R(X,Y)Z = (\frac{r}{2} + 2f^2 + 2f')\{g(Y,Z)X - g(X,Z)Y\}$$

$$- (\frac{r}{2} + 3f^2 + 3f')\{(g(Y,Z)\xi - \eta(Z)Y)\eta(X)$$

$$- (g(X,Z)\xi - \eta(Z)X)\eta(Y)\},$$
(3)

(4)
$$S(X,Y) = (\frac{r}{2} + f^2 + f')g(X,Y) - (\frac{r}{2} + 3f^2 + 3f')\eta(X)\eta(Y),$$

(5)
$$QX = (\frac{r}{2} + f^2 + f')X - (\frac{r}{2} + 3f^2 + 3f')\eta(X)\xi,$$

where r is the scalar curvature of M. Also we get the following equations by using (3), (4) and (5)

(6)
$$R(X,Y)\xi = -(f^2 + f')(\eta(Y)X - \eta(X)Y),$$

(7)
$$S(X,\xi) = -2(f^2 + f')\eta(X),$$

(8)
$$Q\xi = -2(f^2 + f')\xi.$$

As a consequence of (2), we have

(9)
$$(\nabla_X \eta)(Y) = f(g(X, Y) - \eta(X)\eta(Y)),$$

for all vector fields X and Y.

DEFINITION 2.1. [11] A Riemannian manifold M of dimension n is called an η Einstein manifold if

$$S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y),$$

for all vector fields X, Y and a, b are functions on M.

DEFINITION 2.2. [17] Let M be a Riemannian manifold of dimension n with metric g. A tensor field γ of type (0,2) is called parallel tensor if $\nabla \gamma = 0$, where ∇ is the operator of covariant differentiation with respect to the metric tensor g.

3. Conformal Ricci solitons on three-dimensional f-Kenmotsu manifold

Let M be an f-Kenmotsu manifold of dimension three. Then the conformal Ricci soliton is given by

(10)
$$\mathcal{L}_{V}g + 2S = \left[2\lambda - (p + \frac{2}{3})\right]g.$$

Let the potential vector field V be the Reeb vector field ξ , then we get

$$(\mathcal{L}_{\xi}g)(X,Y) = 2f(g(X,Y) - \eta(X)\eta(Y)).$$

Therefore, from (10) and (11), we get

(12)
$$S(X,Y) = Ag(X,Y) + f\eta(X)\eta(Y),$$

where $A = \frac{1}{2}[2\lambda - (p + \frac{2}{3}) - 2f].$

Thus we can state the following

Theorem 3.1. An f-Kenmotsu manifold of dimension three admitting conformal Ricci soliton is an η -Einstein manifold.

From (12), we get

(13)
$$S(\xi, X) = \frac{1}{2} \left[2\lambda - (p + \frac{2}{3}) \right] \eta(X),$$

(14)
$$QX = \frac{1}{2} [2\lambda - (p + \frac{2}{3}) - 2f]X + f\eta(X)\xi,$$

(15)
$$S(\xi,\xi) = \frac{1}{2} [2\lambda - (p + \frac{2}{3})],$$

(16)
$$Q\xi = \frac{1}{2}[2\lambda - (p + \frac{2}{3})]\xi.$$

Let $\{e_i\}$ be an orthonormal basis of the tangent space of the manifold M. Taking $X = Y = e_i$ in (12) and summing over i, we get

(17)
$$r = \frac{3}{2} [2\lambda - (p + \frac{2}{3}) - 2f] + f.$$

But for conformal Ricci soliton, r = -1. Thus from (17), we get

$$-1 = \frac{3}{2}[2\lambda - (p + \frac{2}{3}) - 2f] + f,$$

which gives

$$\lambda = \frac{p}{2} + \frac{2}{3}f.$$

Thus we can state the following

PROPOSITION 3.2. If an f-Kenmotsu manifold of dimension three admits conformal Ricci soliton, then the value of λ is $\frac{p}{2} + \frac{2}{3}f$.

4. ϕ -Ricci symmetric f-Kenmotsu manifold of dimension 3 admitting conformal Ricci soliton

DEFINITION 4.1. [6] An f-Kenmotsu manifold of dimension three is said to be ϕ -Ricci symmetric if

(19)
$$\phi^2((\nabla_X Q)(Y)) = 0,$$

for all vector fields X, Y on M, where Q is the Ricci operator.

From (19), we get

$$-(\nabla_X Q)(Y) + \eta((\nabla_X Q)(Y))\xi = 0.$$

Taking inner product, we obtain

$$-g((\nabla_X Q)(Y), Z) + \eta((\nabla_X Q)(Y))\eta(Z) = 0.$$

Simplifying, we obtain

$$-g(\nabla_X Q(Y), Z) + S(\nabla_X Y, Z) + \eta(\nabla_X Q(Y))\eta(Z)$$

$$- \eta(Q(\nabla_X Y))\eta(Z) = 0.$$

Putting $Y = \xi$ in (20), we get

$$-g(\nabla_X Q(\xi), Z) + S(\nabla_X \xi, Z) + \eta(\nabla_X Q(\xi))\eta(Z)$$

$$- \eta(Q(\nabla_X \xi))\eta(Z) = 0.$$
(21)

Using (14) and (16) in (21), we get

$$(22) -Bg(\nabla_X \xi, Z) + S(\nabla_X \xi, Z) = 0,$$

where $B = \frac{1}{2} [2\lambda - (p + \frac{2}{3})].$

From (2) and (22), we have

(23)
$$-Bf[g(X,Z) - \eta(X)\eta(Z)] + f[S(X,Z) - \eta(X)S(\xi,Z)] = 0.$$

Using (12) and (13) in (23), we get

$$- f^{2}[g(X,Z) - \eta(X)\eta(Z)] = 0.$$

Since, $g(X, Z) - \eta(X)\eta(Z) \neq 0$ for all vector fields X and Z, we get f = 0. Thus we can state the following

THEOREM 4.1. If an f-Kenmotsu manifold of dimension three admitting conformal Ricci soliton is ϕ -Ricci symmetric, then the manifold is cosymplectic.

5. η -parallel Ricci tensor of an f-Kenmotsu manifold admitting conformal Ricci soliton

DEFINITION 5.1. [5] The Ricci tensor S of an f-Kenmotsu manifold of dimension three is called η -parallel if it satisfies

$$(\nabla_W S)(\phi X, \phi Y) = 0,$$

for all vector fields X, Y and W on M.

From (12), we get

(24)
$$S(\phi X, \phi Y) = \frac{1}{2} [2\lambda - (p + \frac{2}{3}) - 2f](g(X, Y) - \eta(X)\eta(Y)).$$

Differentiating (24) covarianty with respect to W, we obtain

$$(\nabla_W S)(\phi X, \phi Y) = -\frac{1}{2} [2\lambda - (p + \frac{2}{3}) - 2f]((\nabla_W \eta)(X)\eta(Y) + \eta(X)(\nabla_W \eta)(Y)).$$
(25)

Using (9) in (25), we get

$$(\nabla_W S)(\phi X, \phi Y) = -\frac{f}{2} [2\lambda - (p + \frac{2}{3}) - 2f][g(X, W)\eta(Y) + g(Y, W)\eta(X) - 2\eta(X)\eta(Y)\eta(W)].$$
(26)

Let the Ricci tensor of the manifold be η -parallel, then from (26), we get

$$\frac{f}{2}[2\lambda - (p + \frac{2}{3}) - 2f][g(X, W)\eta(Y) + g(Y, W)\eta(X) -2\eta(X)\eta(Y)\eta(W)] = 0.$$
(27)

Using (18), we get from (27), either f=0 or f=-1.

Thus we can state the following

Theorem 5.1. If Ricci tensor of a non-cosymplectic f-Kenmotsu manifold of dimension three admitting conformal Ricci soliton is η -parallel, then the manifold is β -Kenmotsu.

6. Cyclic parallel Ricci tensor on an f-Kenmotsu manifold admitting conformal Ricci tensor

DEFINITION 6.1. [5] Ricci tensor S of an f-Kenmotsu manifold M of dimension three is called cyclic-parallel if

$$(\nabla_X S)(Y, Z) + (\nabla_Y S)(Z, X) + (\nabla_Z S)(X, Y) = 0,$$

for all vector fields X, Y, Z on M and ∇ denote the Riemannian connection. From (12), we get

$$(\nabla_X S)(Y, Z) = -f((\nabla_X \eta)(Y)\eta(Z) + \eta(Y)(\nabla_X \eta)(z).$$

Therefore, we have from above

$$(\nabla_{X}S)(Y,Z) + (\nabla_{Y}S)(Z,X) + (\nabla_{Z}S)(X,Y)$$

$$= -f\{(\nabla_{X}\eta)(Y)\eta(Z) + \eta(Y)(\nabla_{X}\eta)(Z)$$

$$+(\nabla_{Y}\eta)(Z)\eta(X) + \eta(Z)(\nabla_{Y}\eta)(X)$$

$$+\nabla_{Z}\eta)(X)\eta(Y) + \eta(X)(\nabla_{Z}\eta)(Y)\}.$$
(28)

Let the Ricci tensor is cyclic-parallel, then from (28), we get

$$f\{(\nabla_X \eta)(Y)\eta(Z) + \eta(Y)(\nabla_X \eta)(Z) + (\nabla_Y \eta)(Z)\eta(X) + \eta(Z)(\nabla_Y \eta)(X) + (\nabla_Z \eta)(X)\eta(Y) + \eta(X)(\nabla_Z \eta)(Y)\} = 0.$$

From (9) and (29), we get

(30)
$$f^{2}[g(X,Y)\eta(Z) + g(Y,Z)\eta(X) + g(Z,X)\eta(Y) - 3\eta(X)\eta(Y)\eta(Z)] = 0.$$

Putting $X = Y = e_i$ in (30), where $\{e_i\}$ (i = 1, 2, 3) is the orthonormal basis of the tangent space of the manifold and summing over i, we get

$$2f^2\eta(Z) = 0,$$

which gives f = 0.

Thus we can state the following

THEOREM 6.1. If the Ricci tensor of an f-Kenmotsu manifold of dimension three admitting conformal Ricci soliton is cyclic-parallel, then the manifold is cosymplectic.

7. Second order parallel tensor and conformal Ricci soliton

Let γ be a second order symmetric tensor field on a regular f-Kenmotsu manifold M of dimension three, that is, $\gamma(X,Y) = \gamma(Y,X)$ such that $\nabla \gamma = 0$. Then from the Ricci identity, we have

$$\nabla^2 \gamma(X, Y; Z, W) = \nabla^2 \gamma(X, Y; W, Z).$$

From above, we obtain

(31)
$$\gamma(R(X,Y)Z,W) + \gamma(R(X,Y)W,Z) = 0,$$

for all vector fields X, Y, Z and W on M.

Substituting $X = Z = W = \xi$ in (31), we get

(32)
$$\gamma(R(\xi, Y)\xi, \xi) = 0.$$

From (6), we get

(33)
$$R(\xi, Y)\xi = -(f^2 + f')(\eta(Y)\xi - Y).$$

Therefore, from (32) and (33), we obtain

$$- (f^{2} + f')(\eta(Y)\gamma(\xi, \xi) - \gamma(Y, \xi)) = 0.$$

Since, $f^2 + f' \neq 0$, we get from above

(34)
$$\gamma(Y,\xi) = \eta(Y)\gamma(\xi,\xi).$$

Taking differentiation (34) covariantly with respect to X, we get

(35)
$$\gamma(\nabla_X Y, \xi) + \gamma(Y, \nabla_X \xi) = g(\nabla_X Y, \xi) \gamma(\xi, \xi) + (Y, \nabla_X \xi) \gamma(\xi, \xi) + 2\eta(Y) \gamma(\nabla_X \xi, \xi).$$

From (34), we obtain

(36)
$$\gamma(\nabla_X Y, \xi) = g(\nabla_X Y, \xi)\gamma(\xi, \xi).$$

From (35) and (36), we get

(37)
$$\gamma(Y, \nabla_X \xi) = g(Y, \nabla_X \xi) \gamma(\xi, \xi) + 2\eta(Y) \gamma(\nabla_X \xi, \xi).$$

From (2) and (37), we get

(38)
$$f[g(X,Y)\gamma(\xi,\xi) - 3\eta(X)\eta(Y)\gamma(\xi,\xi) + 2\eta(Y)\gamma(X,\xi) - \gamma(X,Y) + \eta(X)\gamma(\xi,Y)] = 0.$$

From (34), we get

(39)
$$\eta(X)\gamma(Y,\xi) = \eta(X)\eta(Y)\gamma(\xi,\xi).$$

From (38) and (39), we obtain

$$f[g(X,Y)\gamma(\xi,\xi) - \gamma(X,Y)] = 0.$$

Since $f \neq 0$, we get from above

$$\gamma(X,Y) = q(X,Y)\gamma(\xi,\xi).$$

Thus we can state

THEOREM 7.1. If γ is a second order parallel tensor of a regular f-Kenmotsu manifold of dimension three, then γ is given by

$$\gamma(X,Y) = g(X,Y)\gamma(\xi,\xi),$$

for all vector fields X, Y on M.

Since $\nabla\{[2\lambda - (p + \frac{2}{3})]g(X,Y)\} = 0$ for all vector fields X and Y, we can say that $\{\mathcal{L}_{\xi}g(X,Y) + 2S(X,Y)\}$ is a second order parallel tensor. So,

(40)
$$(\mathcal{L}_{\xi}g)(X,Y) + 2S(X,Y) = \{ (\mathcal{L}_{\xi}g)(\xi,\xi) + 2S(\xi,\xi) \} g(X,Y).$$

From (7) and (11), we get

(41)
$$(\mathcal{L}_{\xi}g)(\xi,\xi) + 2S(\xi,\xi) = -2(f^2 + f').$$

From (40) and (41), we obtain

(42)
$$(\mathcal{L}_{\xi}g)(X,Y) + 2S(X,Y) = -2(f^2 + f')g(X,Y).$$

Compairing (10) and (42), we get

(43)
$$\lambda = -(f^2 + f') + \frac{1}{2}(p + \frac{2}{3}).$$

From (18) and (43), we get

$$3(f^2 + f') + 2f = 1.$$

Thus we can state the following

PROPOSITION 7.2. If a regular f-Kenmotsu manifold of dimension three admits conformal Ricci soliton, then $3(f^2 + f') + 2f = 1$.

8. Example

Let us consider the manifold $M=\{x,y,z\in\mathbb{R}^3:z\neq 0\}$ of dimension 3, where (x,y,z) are standard co-ordinates in \mathbb{R}^3 . We choose the vector fields

$$e_1 = z^{-2} \frac{\partial}{\partial x}, \quad e_2 = z^{-2} \frac{\partial}{\partial y}, \quad e_3 = \frac{\partial}{\partial z},$$

which are linearly independent at each point of M. We get the following by direct computations

$$[e_1, e_2] = 0, \quad [e_1, e_3] = -\frac{2}{z}e_1, \quad [e_2, e_3] = -\frac{2}{z}e_2.$$

Let the metric tensor q be defined by

$$g(e_1, e_1) = g(e_2, e_2) = g(e_3, e_3) = 1$$

and

$$q(e_1, e_2) = q(e_1, e_3) = q(e_2, e_3) = 0.$$

The 1-form η is defined by $\eta(X) = g(X, e_3)$, for all X on M. Let ϕ be the (1, 1)-tensor field defined by

$$\phi(e_1) = e_2, \quad \phi(e_2) = -e_1, \quad \phi(e_3) = 0.$$

Then we find that

$$\eta(e_3) = 1, \quad \phi^2 X = -X + \eta(X)e_3,
g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y),$$

for any vector fields X, Y on M.

Let ∇ be the Levi-Civita connection on M, then by Koszul'S formula, we obtain

$$\nabla_{e_1} e_1 = \frac{2}{z} e_3, \quad \nabla_{e_1} e_2 = 0, \quad \nabla_{e_1} e_3 = -\frac{2}{z} e_1,$$

$$\nabla_{e_2} e_2 = \frac{2}{z} e_3, \quad \nabla_{e_2} e_1 = 0, \quad \nabla_{e_2} e_3 = -\frac{2}{z} e_2,$$

$$\nabla_{e_3} e_3 = 0, \quad \nabla_{e_3} e_1 = 0, \quad \nabla_{e_3} e_2 = 0.$$

From the above expressions of ∇ , the manifold is an f-Kenmotsu manifold of dimension three with $\xi = e_3$ and $f = -\frac{2}{z}$. Also $f^2 + f' \neq 0$, thus the manifold is a regular f-Kenmotsu manifold.

Using the formula $R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z$, we get

$$\begin{split} R(e_1,e_2)e_2 &= -\frac{4}{z^2}e_1, \quad R(e_2,e_1)e_1 = -\frac{4}{z^2}e_2, \quad R(e_2,e_3)e_3 = -\frac{6}{z^2}e_2, \\ R(e_3,e_2)e_2 &= -\frac{6}{z^2}e_3, \quad R(e_1,e_3)e_3 = -\frac{6}{z^2}e_1, \quad R(e_3,e_1)e_1 = -\frac{6}{z^2}e_3, \\ R(e_1,e_2)e_3 &= 0, \quad R(e_2,e_3)e_1 = 0, \quad R(e_3,e_1)e_2 = 0. \end{split}$$

From the above expressions of curvature tensor, we get

$$S(e_1, e_1) = -\frac{10}{z^2}, \quad S(e_2, e_2) = -\frac{10}{z^2}, \quad S(e_3, e_3) = -\frac{12}{z^2}.$$

Let r be the scalar curvature, then from above

$$r = S(e_1, e_1) + S(e_2, e_2) + S(e_3, e_3) = -\frac{32}{z^2}.$$

But for conformal Ricci soliton r = -1. Thus the given manifold admits conformal Ricci soliton if $z = 4\sqrt{2}$.

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