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CONVERGENCE OF AN ITERATIVE ALGORITHM FOR SYSTEMS OF GENERALIZED VARIATIONAL INEQUALITIES

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Abstract. In this paper, we introduce and consider a new system of generalized variational inequalities involving five different operators. Using the sunny nonexpansive retraction technique we suggest and analyze some new explicit iterative methods for this system of variational inequalities. We also study the convergence analysis of the new iterative method under certain mild conditions. Our results can be viewed as a refinement and improvement of the previously known results for variational inequalities.

1. Introduction

Let $(E, \|\cdot\|)$ be a Banach space and C be a nonempty closed convex subset of E. This paper deals with the problem of convergence of an iterative algorithm for a system of generalized variational inequalities in a Banach space: Find $(x^*, y^*) \in C \times C$ such that

$$(1.1) \qquad \begin{cases} \langle \rho A_1(y^*) + x^* - g_1(y^*), J(g_1(x) - x^*)\rangle \geq 0, & \forall x \in C, \\ \langle \eta A_2(x^*) + y^* - g_2(x^*), J(g_2(x) - y^*)\rangle \geq 0, & \forall x \in C, \end{cases}$$

where $A_i, g_i : C \to E$ are four nonlinear mappings for $i = 1, 2$, J is the normalized duality mapping and $\rho, \eta > 0$ are positive constants.

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If $E = H$ is a real Hilbert space, $g_1 = g_2 = I$(I denotes the identity operator), then problem (1.1) reduces to the following general system of variational inequalities in a Hilbert space: Find $(x^*, y^*) \in C \times C$ such that

$$\text{(1.2)} \qquad \begin{cases} \langle \rho A_1(y^*) + x^* - y^*, x - x^* \rangle \geq 0, & \forall x \in C, \\ \langle \eta A_2(x^*) + y^* - x^*, x - y^* \rangle \geq 0, & \forall x \in C, \end{cases}$$

which was considered by Ceng et al. [1]. In particular, if $A_1 = A_2 = A$, then problem (1.2) reduces to the following system of variational inequalities: Find $(x^*, y^*) \in C \times C$ such that

$$\text{(1.3)} \qquad \begin{cases} \langle \rho A(y^*) + x^* - y^*, x - x^* \rangle \geq 0, & \forall x \in C, \\ \langle \eta A(x^*) + y^* - x^*, x - y^* \rangle \geq 0, & \forall x \in C, \end{cases}$$

which was defined and studied by Verma [5]. Further, if $x^* = y^*$, then problem (1.3) reduces to the following classical variational inequality (VI(A, C)): Find $x^* \in C$ such that

$$\text{(1.4)} \qquad \langle A(x^*), y - x^* \rangle \geq 0, \quad \forall y \in C.$$

We can see easily that the variational inequality (1.4) is equivalent to a fixed point problem. An element $x^* \in C$ is a solution of the variational inequality (1.4) if and only if $x^* \in C$ is a fixed point of the mapping $P_C(I - \lambda A)$, where I is the identity mapping, $\lambda > 0$ is a constant and P_C is the metric projection of H onto C. This alternative equivalent formulation has played a significant role in the studies of the variational inequalities and related optimization problem.

Furthermore, in order to solve the VI(A, C) (1.4) in the Euclidean space $\mathbb{R}^n$, Korpelevich [3] introduced the following so-called extra-gradient method:

$$\begin{cases} x_0 = x \in C, \\ y_n = P_C(x_n - \lambda A(x_n)), \\ x_{n+1} = P_C(x_n - \lambda A(y_n)), & \forall n \geq 0, \end{cases}$$

where $\lambda > 0$ is a constant.

We know that P_C is a firmly nonexpansive mapping of H onto C, i.e.,

$$\langle x - y, P_C(x) - P_C(y) \rangle \geq \|P_C(x) - P_C(y)\|^2, \quad \forall x, y \in H.$$

It is also known that P_C is characterized by the following property:

$$\langle x - P_C(x), y - P_C(y)\rangle \leq 0, \quad \forall x \in H, y \in C.$$

The content of this paper is organized as follows. In section 2, we present some basic definitions and results frequently used in the content of the approximation solvability of nonlinear variational inequality problems based on iterative procedures. Section 3 is devoted to establishing the main result of this paper. Firstly, using the sunny nonexpansive retraction technique we give some new explicit iterative methods for this system of variational inequalities. Secondly, we show that the convergence analysis of the new iterative method under certain mild conditions.

2. Preliminaries

Let C be a nonempty closed convex subset of a Banach space E with dual space E^*, $\langle \cdot, \cdot\rangle$ be the dual pair between E and E^*, 2^E denote the family of all the nonempty subsets of E. The generalized duality mapping $J_q : E \to 2^{E^*}$ is defined by

$$J_q(x) = \{f^* \in E^* : \langle x, f^*\rangle = \|x\|^q, \|f^*\| = \|x\|^{q-1}\}, \quad \forall x \in E,$$

where $q > 1$ is a constant. In particular, $J = J_2$ is the usual normalized duality mapping. It is known that, in general, $J_q(x) = \|x\|^{q-2}J(x)$ for all $x \neq 0$ and J_q is single-valued if E^* is strictly convex. If E is a Hilbert space, then $J = I$, where I is the identity mapping. In this paper, we use J to denote the single-valued normalized duality mapping.

Let $U = \{x \in E : \|x\| = 1\}$. A Banach space E is said to be uniformly convex if for each $\varepsilon \in (0, 2]$, there exists $\delta > 0$ such that for any $x, y \in U$, $\|x - y\| \geq \varepsilon$ implies $\|\frac{x+y}{2}\| \leq 1 - \delta$. It is known that a uniformly convex Banach space is reflexive and strictly convex. A Banach space E is said to be smooth if the limit

$$\lim_{t\to 0} \frac{\|x + ty\| - \|x\|}{t}$$

exists for all $x, y \in U$. The modulus of smoothness of E is the function $\rho_E : [0,\infty) \to [0,\infty)$ defined by

$$\rho_E(t) = \sup\{\frac{1}{2}(\|x + y\| + \|x - y\|) - 1 : \|x\| \leq 1, \|y\| \leq t\}.$$

A Banach space E is called uniformly smooth if

$$\lim_{t\to 0}\frac{\rho_E(t)}{t}=0.$$

E is called q-uniformly smooth if there exists a constant $c>0$ such that

$$\rho_E(t)\le ct^q,\quad q>1.$$

If E is q-uniformly smooth, then $q\le 2$ and E is uniformly smooth.

Definition 2.1. Let $A:E\to E$ be a single-valued mapping. A is said to be

(i) accretive if

$$\langle A(x)-A(y),J(x-y)\rangle\ge 0,\quad \forall x,y\in E;$$

(ii) r-strongly accretive if there exists a constant $r>0$ such that

$$\langle A(x)-A(y),J(x-y)\rangle\ge r\|x-y\|^2,\quad \forall x,y\in E;$$

(iii) m-relaxed cocoersive if there exists a constant $m>0$ such that

$$\langle A(x)-A(y),J(x-y)\rangle\ge -m\|x-y\|^2,\quad \forall x,y\in E;$$

(iv) (α,ξ)-relaxed cocoercive if there exist constants $\alpha,\xi>0$ such that

$$\langle A(x)-A(y),J(x-y)\rangle\ge -\alpha\|A(x)-A(y)\|^2+\xi\|x-y\|^2,\ \forall x,y\in E.$$

Let D be a subset of C and Q be a mapping of C into D. Then Q is said to be sunny if $Q[Q(x)+t(x-Q(x))]=Q(x)$, whenever $Q(x)+t(x-Q(x))\in C$ for $x\in C$ and $t\ge 0$. A mapping Q of C into itself is called a retraction if $Q^2=Q$. If a mapping Q of C into itself is a retraction, then $Q(z)=z$ for all $z\in R(Q)$, where $R(Q)$ is the range of Q. A subset D of C is called a sunny nonexpansive retract of C if there exists a sunny nonexpansive retraction from C onto D.

In order to prove our main results, we also need the following lemmas.

Lemma 2.1([7]). Let E be a real 2-uniformly smooth Banach space. Then

$$\|x+y\|^2\le\|x\|^2+2\langle y,J(x)\rangle+2\|Ky\|^2,\quad \forall x,y\in E,$$

where J is the normalized duality mapping and K is the 2-uniformly smooth constant of E.

Lemma 2.2([4]). Let C be a nonempty closed convex subset of a smooth Banach space E and let Q_C be a retraction from E onto C. Then the following are equivalent:

(i) Q_C is both sunny and nonexpansive;

(ii) $\langle x - Q_C(x), J(y - Q_C(x))\rangle \le 0$ for all $x \in E$ and $y \in C$.

Lemma 2.3([6]). Suppose $\{\delta_n\}_{n=0}^{\infty}$ is a nonnegative sequence satisfying the following inequality:

$$\delta_{n+1} \le (1-\lambda_n)\delta_n + \sigma_n, \quad \forall n \ge 0,$$

with $\lambda_n \in [0,1]$, $\sum_{n=0}^{\infty} \lambda_n = \infty$ and $\sigma_n = 0(\lambda_n)$. Then $\lim_{n\to\infty} \delta_n = 0$.

3. Main results

In this section, we consider the convergence criteria of explicit iterative algorithm under some suitable mild conditions.

Theorem 3.1. Let C be a nonempty closed convex subset of a smooth Banach space E. Let $Q_C : E \to C$ be a sunny nonexpansive retraction, $A_i, g_i : C \to E$ be four single-valued mappings for $i = 1, 2$. Then (x^*, y^*) with $x^*, y^* \in C$ is a solution of problem (1.1) if and only if

$$\begin{cases} x^* &= Q_C[g_1(y^*) - \rho A_1(y^*)], \\ y^* &= Q_C[g_2(x^*) - \eta A_2(x^*)]. \end{cases} \tag{3.1}$$

Proof. Applying Lemma 2.2, we have that

$$\begin{cases} \langle \rho A_1(y^*) + x^* - g_1(y^*), J(g_1(x) - x^*)\rangle & \ge 0, \quad \forall x \in C, \\ \langle \eta A_2(x^*) + y^* - g_2(x^*), J(g_2(x) - y^*)\rangle & \ge 0. \quad \forall x \in C. \end{cases}$$

$$\Updownarrow$$

$$\begin{cases} \langle g_1(y^*) - \rho A_1(y^*) - x^*, J(g_1(x) - x^*)\rangle & \le 0, \quad \forall x \in C, \\ \langle g_2(x^*) - \eta A_2(x^*) - y^*, J(g_2(x) - y^*)\rangle & \le 0, \quad \forall x \in C. \end{cases}$$

$$\Updownarrow$$

$$\begin{cases} x^* &= Q_C[g_1(y^*) - \rho A_1(y^*)], \\ y^* &= Q_C[g_2(x^*) - \eta A_2(x^*)]. \end{cases}$$

Corollary 3.1. Let C be a nonempty closed convex subset of a Hilbert space H. Let P_C be the metric projection and $A_1, A_2 : C \to H$

be single-valued mappings. Then (x^*, y^*) with $x^*, y^* \in C$ is a solution of problem (1.2) if and only if

$$\begin{cases} x^* = P_C[y^* - \rho A_1(y^*)], \\ y^* = P_C[x^* - \eta A_2(x^*)]. \end{cases}$$

Theorem 3.2. Let E be a uniformly convex and 2-uniformly smooth Banach space with the 2-uniformly smooth constant K, C be a nonempty closed convex subset of E and Q_C be a sunny nonexpansive retraction from E onto C. Let $A_i, g_i : C \to E$ be mappings such that A_i is (ω_i, ξ_i)-relaxed cocoercive, μ_i-Lipschitz continuous and g_i is r_i-strongly accretive, λ_i-Lipschitz continuous for $i = 1, 2$. Let (x^*, y^*) be the solution of problem (1.1). For fixed $x_0, y_0 \in C$ arbitrarily, let $\{x_n\}$, $\{y_n\} \subset C$ be sequences generated by

$$\begin{cases} x_{n+1} &= (1-\alpha_n)x_n + \alpha_n Q_C[g_1(y_n) - \rho A_1(y_n)], \\ y_{n+1} &= Q_C[g_2(x_{n+1}) - \eta A_2(x_{n+1})], \end{cases} \tag{3.2}$$

where $\{\alpha_n\}$ is a real number sequence in $[0, 1]$ satisfying the following restriction $\sum_{n=0}^{\infty} \alpha_n = \infty$. If

$$\left| \rho - \frac{\xi_1 - \omega_1\mu_1^2}{2K^2\mu_1^2} \right| < \frac{\sqrt{(\xi_1 - \omega_1\mu_1^2)^2 - 2K^2\mu_1^2 m_1(2-m_1)}}{2K^2\mu_1^2}, \tag{3.3}$$

$$\left| \eta - \frac{\xi_2 - \omega_2\mu_2^2}{2K^2\mu_2^2} \right| < \frac{\sqrt{(\xi_2 - \omega_2\mu_2^2)^2 - 2K^2\mu_2^2 m_2(2-m_2)}}{2K^2\mu_2^2}, \tag{3.4}$$

$$\xi_1 > \omega_1\mu_1^2 + \mu_1 K\sqrt{2m_1(2-m_1)}, \quad m_1 < 1, \tag{3.5}$$

$$\xi_2 > \omega_2\mu_2^2 + \mu_2 K\sqrt{2m_2(2-m_2)}, \quad m_2 < 1, \tag{3.6}$$

where

$$m_1 = \sqrt{1 - 2r_1 + 2K^2\lambda_1^2},$$

$$m_2 = \sqrt{1 - 2r_2 + 2K^2\lambda_2^2},$$

then $\{x_n\}$, $\{y_n\}$ defined by (3.2) converge strongly to x^*, y^*, respectively.

Proof. To prove the result we need first to evaluate $\|x_{n+1} - x^*\|$ for all $n \geq 0$.

From (3.1), (3.2) and the nonexpansive property of the sunny nonexpansive retraction Q_C, we have

$$
\begin{aligned}
\|x_{n+1} - x^*\| &\le (1-\alpha_n)\|x_n - x^*\| \\
&\qquad + \alpha_n \|Q_C[g_1(y_n) - \rho A_1(y_n)] - Q_C[g_1(y^*) - \rho A_1(y^*)]\| \\
&\le (1-\alpha_n)\|x_n - x^*\| \\
&\qquad + \alpha_n \|g_1(y_n) - g_1(y^*) - \rho[A_1(y_n) - A_1(y^*)]\| \\
&\le (1-\alpha_n)\|x_n - x^*\| + \alpha_n\|y_n - y^* - \rho[A_1(y_n) - A_1(y^*)]\| \\
&\qquad + \alpha_n \|y_n - y^* - (g_1(y_n) - g_1(y^*))\|. \qquad (3.7)
\end{aligned}
$$

Since $A_1 : C \to E$ is (ω_1, ξ_1)-relaxed cocoercive and μ_1-Lipschitz continuous, we have

$$
\begin{aligned}
&\|y_n - y^* - \rho[A_1(y_n) - A_1(y^*)]\|^2 \\
&\le \|y_n - y^*\|^2 - 2\rho\langle A_1(y_n) - A_1(y^*), J(y_n - y^*)\rangle \\
&\quad + 2\rho^2 K^2 \|A_1(y_n) - A_1(y^*)\|^2 \\
&\le \|y_n - y^*\|^2 - 2\rho[-\omega_1\|A_1(y_n) - A_1(y^*)\|^2 + \xi_1\|y_n - y^*\|^2] \\
&\quad + 2\rho^2 K^2 \|A_1(y_n) - A_1(y^*)\|^2 \\
&\le [1 + 2\rho\omega_1\mu_1^2 - 2\rho\xi_1 + 2\rho^2K^2\mu_1^2]\|y_n - y^*\|^2. \qquad (3.8)
\end{aligned}
$$

Since $g_1 : C \to E$ is r_1-strongly accretive and λ_1-Lipschitz continuous, we have

$$
\begin{aligned}
&\|y_n - y^* - (g_1(y_n) - g_2(y^*))\|^2 \\
&\le \|y_n - y^*\| - 2\langle g_1(y_n) - g_1(y^*), J(y_n - y^*)\rangle + 2K^2\|g_1(y_n) - g_1(y^*)\|^2 \\
&\le \|y_n - y^*\|^2 - 2r_1\|y_n - y^*\|^2 + 2K^2\lambda_1^2\|y_n - y^*\|^2 \\
&\le [1 - 2r_1 + 2K^2\lambda_1^2]\|y_n - y^*\|^2,
\end{aligned}
$$

which implies that

$$
\|y_n - y^* - (g_1(y_n) - g_1(y^*))\| \le m_1\|y_n - y^*\|, \qquad (3.9)
$$

where $m_1 = \sqrt{1 - 2r_1 + 2K^2\lambda_1^2}$. It follows from (3.7)-(3.9) that

$$
\begin{aligned}
\|x_{n+1} - x^*\| &\le (1-\alpha_n)\|x_n - x^*\| \\
&\qquad + \alpha_n\left(\sqrt{1 + 2\rho\omega_1\mu_1^2 - 2\rho\xi_1 + 2\rho^2K^2\mu_1^2} + m_1\right)\|y_n - y^*\| \\
&= (1-\alpha_n)\|x_n - x^*\| + \alpha_n\theta_1\|y_n - y^*\|, \qquad (3.10)
\end{aligned}
$$

where $\theta_1 = \sqrt{1 + 2\rho\omega_1\mu_1^2 - 2\rho\xi_1 + 2\rho^2K^2\mu_1^2} + m_1$.

Next, we estimate $\|y_{n+1} - y^*\|$. From (3.1) and (3.2), we have

$$\begin{aligned}\|y_{n+1} - y^*\| &= \|Q_C[g_2(x_{n+1}) - \eta A_2(x_{n+1})] - Q_C[g_2(x^*) - \eta A_2(x^*)]\| \\ &\le \|x_{n+1} - x^* - \eta[A_2(x_{n+1}) - A_2(x^*)]\| \\ &\quad + \|x_{n+1} - x^* - (g_2(x_{n+1}) - g_2(x^*))\|. \end{aligned} \tag{3.11}$$

Since $A_2 : C \to E$ is (ω_2, ξ_2)-relaxed cocoercive and μ_2-Lipschitz continuous, we have

$$\begin{aligned}&\|x_{n+1} - x^* - \eta[A_2(x_{n+1}) - A_2(x^*)]\|^2 \\ &\le \|x_{n+1} - x^*\|^2 - 2\eta\langle A_2(x_{n+1}) - A_2(x^*), J(x_{n+1} - x^*)\rangle \\ &\quad + 2\eta^2K^2\|A_2(x_{n+1}) - A_2(x^*)\|^2 \\ &\le \|x_{n+1} - x^*\|^2 - 2\eta[-\omega_2\|A_2(x_{n+1}) - A_2(x^*)\|^2 + \xi_2\|x_{n+1} - x^*\|^2] \\ &\quad + 2\eta^2K^2\|A_2(x_{n+1}) - A_2(x^*)\|^2 \\ &\le (1 + 2\eta\omega_2\mu_2^2 - 2\eta\xi_2 + 2\eta^2K^2\mu_2^2)\|x_{n+1} - x^*\|^2. \end{aligned} \tag{3.12}$$

Since $g_2 : C \to E$ is r_2-strongly accretive and λ_2-Lipschitz continuous, we have

$$\begin{aligned}&\|x_{n+1} - x^* - (g_2(x_{n+1}) - g_2(x^*))\|^2 \\ &\le \|x_{n+1} - x^*\|^2 - 2\langle g_2(x_{n+1}) - g_2(x^*), J(x_{n+1} - x^*)\rangle \\ &\quad + 2K^2\|g_2(x_{n+1}) - g_2(x^*)\|^2 \\ &\le \|x_{n+1} - x^*\|^2 - 2r_2\|x_{n+1} - x^*\|^2 + 2K^2\lambda_2\|x_{n+1} - x^*\|^2 \\ &\le [1 - 2r_2 + 2K^2\lambda_2^2]\|x_{n+1} - x^*\|^2, \end{aligned}$$

which implies that

$$\|x_{n+1} - x^* - (g_2(x_{n+1}) - g_2(x^*))\| \le m_2\|x_{n+1} - x^*\|, \tag{3.13}$$

where $m_2 = \sqrt{1 - 2r_2 + 2K^2\lambda_2^2}$. So, from (3.11)-(3.13), it follows that

$$\begin{aligned}\|y_{n+1} - y^*\| &\le \left[\sqrt{1 + 2\eta\omega_2\mu_2^2 - 2\eta\xi_2 + 2\eta^2K^2\mu_2^2} + m_2\right]\|x_{n+1} - x^*\| \\ &= \theta_2\|x_{n+1} - x^*\|, \end{aligned} \tag{3.14}$$

where $\theta_2 = \sqrt{1 + 2\eta\omega_2\mu_2^2 - 2\eta\xi_2 + 2\eta K^2\mu_2^2} + m_2$. By (3.3)-(3.6), we know that $0 \leq \theta_1, \theta_2 < 1$. It follows from (3.10) and (3.14) that

$$\begin{aligned}\|x_{n+1} - x^*\| &\leq (1 - \alpha_n)\|x_n - x^*\| + \alpha_n\theta_1\|y_n - y^*\| \\ &\leq [1 - \alpha_n(1 - \theta_1\theta_2)]\|x_n - x^*\|.\end{aligned}$$

It is clear that $1 - \theta_1\theta_2 \in (0, 1]$ and $\sum_{n=0}^{\infty} \alpha_n(1 - \theta_1\theta_2) = \infty$. Hence, applying Lemma 2.3 to the last inequality, we immediately obtain that $x_n \to x^*$ as $n \to \infty$. And by (3.14), we obtain $y_n \to y^*$ as $n \to \infty$. This completes the proof.

Remark 3.1. (i) We note that Hilbert spaces and $L^p (p \geq 2)$ spaces are uniformly convex and 2-uniformly smooth.

(ii) It is well known that if E is a Hilbert space, then a sunny nonexpansive retraction Q_C is coincident with the metric projection P_C.

If E is a Hilbert space, then the 2-uniformly smooth constant $K = \frac{\sqrt{2}}{2}$. The following result can be deduced from Theorem 3.2 immediately.

Corollary 3.2. Let C be a nonempty closed convex subset of a real Hilbert space H. Let $A_i, g_i : C \to H$ be mappings such that A_i is (ω_i, ξ_i)-relaxed cocoercive, μ_i-Lipschitz continuous and g_i is r_i-strongly monotone, λ_i-Lipschitz continuous for $i = 1, 2$. Let (x^*, y^*) be the solution of problem (1.2). For fixed $x_0, y_0 \in C$ arbitrarily, let $\{x_n\}, \{y_n\} \subset C$ be sequences generated by

$$\begin{cases} x_{n+1} = (1 - \alpha_n)x_n + \alpha_n P_C[y_n - \rho A_1(y_n)], \\ y_{n+1} = P_C[x_{n+1} - \eta A_2(x_{n+1})], \end{cases} \tag{3.15}$$

where $\{\alpha_n\}$ is a real number sequence in $[0, 1]$ satisfying the following restriction $\sum_{n=0}^{\infty} \alpha_n = \infty$. If

$$\left|\rho - \frac{\xi_1 - \omega_1\mu_1^2}{\mu_1^2}\right| < \frac{\xi_1 - \omega_1\mu_1^2}{\mu_1^2},$$

$$\left|\eta - \frac{\xi_2 - \omega_2\mu_2^2}{\mu_2^2}\right| < \frac{\xi_2 - \omega_2\mu_2^2}{\mu_2^2},$$

$$\xi_1 > \omega_1\mu_1^2, \quad \xi_2 > \omega_2\mu_2^2,$$

then $\{x_n\}$, $\{y_n\}$ defined by (3.15) converge strongly to x^*, y^*, respectively.

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