Korean J. Math. **22** (2014), No. 4, pp. 659–670
http://dx.doi.org/10.11568/kjm.2014.22.4.659

FUZZY ALGEBRA HOMOMORPHISMS AND FUZZY DERIVATIONS

Jung Rye Lee

Abstract. In this paper, we prove the Hyers-Ulam stability of homomorphisms in fuzzy Banach algebras and of derivations on fuzzy Banach algebras associated to the Cauchy-Jensen functional equation.

1. Introduction and preliminaries

The theory of fuzzy space has much progressed as developing the theory of randomness. Some mathematicians have defined fuzzy norms on a vector space from various points of view [2, 18, 24, 26, 29, 39]. Following Cheng and Mordeson [7], Bag and Samanta [2] gave an idea of fuzzy norm in such a manner that the corresponding fuzzy metric is of Kramosil and Michalek type [25] and investigated some properties of fuzzy normed spaces [3].

We use the definition of fuzzy normed spaces given in [2, 29, 30] to investigate a fuzzy version of the Hyers-Ulam stability for the Cauchy-Jensen functional equation in the fuzzy normed algebra setting.

Definition 1.1. [2, 29–31] Let X be a real vector space. A function $N : X \times \mathbb{R} \to [0, 1]$ is called a *fuzzy norm* on X if for all $x, y \in X$ and all $s, t \in \mathbb{R}$,

Received November 21, 2014. Revised November 26, 2014. Accepted November 26, 2014.

2010 Mathematics Subject Classification: 39B52, 46S40, 47S40, 47B47, 17B40.

Key words and phrases: fuzzy Banach algebra; fuzzy algebra homomorphism; fuzzy derivation; Hyers-Ulam stability; Cauchy-Jensen functional equation.

(N_1) $N(x,t)=0$ for $t\leq 0$;
(N_2) $x=0$ if and only if $N(x,t)=1$ for all $t>0$;
(N_3) $N(cx,t)=N(x,\frac{t}{|c|})$ if $c\neq 0$;
(N_4) $N(x+y,s+t)\geq \min\{N(x,s),N(y,t)\}$;
(N_5) $N(x,\cdot)$ is a non-decreasing function of $\mathbb{R}$ and $\lim_{t\to\infty} N(x,t)=1$;
(N_6) for $x\neq 0$, $N(x,\cdot)$ is continuous on $\mathbb{R}$.

The pair (X,N) is called a *fuzzy normed vector space.*

Definition 1.2. [2, 29–31] (1) Let (X,N) be a fuzzy normed vector space. A sequence $\{x_n\}$ in X is said to *be convergent* or *converge* if there exists an $x\in X$ such that $\lim_{n\to\infty} N(x_n-x,t)=1$ for all $t>0$. In this case, x is called the *limit* of the sequence $\{x_n\}$ and we denote it by N-$\lim_{n\to\infty} x_n=x$.

(2) Let (X,N) be a fuzzy normed vector space. A sequence $\{x_n\}$ in X is called *Cauchy* if for each $\varepsilon>0$ and each $t>0$ there exists an $n_0\in\mathbb{N}$ such that for all $n\geq n_0$ and all $p>0$, we have $N(x_{n+p}-x_n,t)>1-\varepsilon$.

It is well-known that every convergent sequence in a fuzzy normed vector space is Cauchy. If each Cauchy sequence is convergent, then the fuzzy norm is said to be *complete* and the fuzzy normed vector space is called a *fuzzy Banach space*.

We say that a mapping $f:X\to Y$ between fuzzy normed vector spaces X and Y is continuous at a point $x_0\in X$ if for each sequence $\{x_n\}$ converging to x_0 in X, then the sequence $\{f(x_n)\}$ converges to $f(x_0)$. If $f:X\to Y$ is continuous at each $x\in X$, then $f:X\to Y$ is said to be *continuous* on X (see [3]).

Definition 1.3. Let X be an algebra and (X,N) a fuzzy normed space.

(1) The fuzzy normed space (X,N) is called a *fuzzy normed algebra* if

$$N(xy,st)\geq N(x,s)\cdot N(y,t)$$

for all $x,y\in X$ and all positive real numbers s and t.

(2) A complete fuzzy normed algebra is called a *fuzzy Banach algebra.*

Example 1.4. Let $(X,\ \|\cdot\|)$ be a normed algebra. Let

$$N(x,t)=\begin{cases}\frac{t}{t+\|x\|} & t>0, x\in X\\ 0 & t\leq 0, x\in X.\end{cases}$$

Then $N(x,t)$ is a fuzzy norm on X and $(X,N(x,t))$ is a fuzzy normed algebra.

Definition 1.5. Let (X, N_X) and (Y, N) be fuzzy normed algebras. Then a multiplicative $\mathbb{R}$-linear mapping $H : (X, N_X) \to (Y, N)$ is called a *fuzzy algebra homomorphism*.

Definition 1.6. Let (Y, N) be a fuzzy normed algebra. Then an $\mathbb{R}$-linear mapping $D : (Y, N) \to (Y, N)$ is called a *fuzzy derivation* if $D(xy) = D(x)y + xD(y)$ for all $x, y \in Y$.

The stability problem of functional equations originated from a question of Ulam [38] concerning the stability of group homomorphisms. Hyers [20] gave a first affirmative partial answer to the question of Ulam for Banach spaces. Hyers' Theorem was generalized by Aoki [1] for additive mappings and by Th.M. Rassias [35] for linear mappings by considering an unbounded Cauchy difference. A generalization of the Th.M. Rassias theorem was obtained by Găvruta [19] by replacing the unbounded Cauchy difference by a general control function in the spirit of Th.M. Rassias' approach.

The functional equation $f(x + y) + f(x - y) = 2f(x) + 2f(y)$ is called a quadratic functional equation. The Hyers-Ulam stability of the quadratic functional equation was proved by Skof [37] for mappings $f : X \to Y$, where X is a normed space and Y is a Banach space. Cholewa [9] noticed that the theorem of Skof is still true if the relevant domain X is replaced by an Abelian group. Czerwik [10] proved the Hyers-Ulam stability of the quadratic functional equation. The stability problems of several functional equations have been extensively investigated by a number of authors and there are many interesting results concerning this problem (see [8, 11, 12], [14]– [17], [21, 23]).

In 1996, G. Isac and Th.M. Rassias [22] were the first to provide applications of stability theory of functional equations for the proof of new fixed point theorems with applications. By using fixed point methods, the stability problems of several functional equations have been extensively investigated by a number of authors (see [5, 6, 28], [32]– [34]).

In this paper, we prove the Hyers-Ulam stability of homomorphisms and derivations in fuzzy Banach algebras associated with the Cauchy-Jensen functional equation.

Throughout this paper, assume that (X, N_X) is a fuzzy normed algebra and that (Y, N) is a fuzzy Banach algebra.

2. Hyers-Ulam stability of homomorphisms in fuzzy Banach algebras

In this section, using the direct method, we prove the Hyers-Ulam stability of homomorphisms in fuzzy Banach algebras associated with the Cauchy-Jensen functional equation.

THEOREM 2.1. *Let $\varphi : X^3 \to [0, \infty)$ be a function such that*

$$\widetilde{\varphi}(x, y, z) := \sum_{j=1}^{\infty} 2^j \varphi\left(\frac{x}{2^j}, \frac{y}{2^j}, \frac{z}{2^j}\right) < \infty \tag{1}$$

for all $x, y, z \in X$. Let $f : X \to Y$ be a mapping satisfying $f(0) = 0$ and

$$\lim_{t\to\infty} N\left(2f\left(\frac{rx + ry}{2} + rz\right) - rf(x) - rf(y) - 2rf(z), t\varphi(x, y, z)\right) = 1 \tag{2}$$

uniformly on X^3 for each $r \in \mathbb{R}$, and

$$\lim_{t\to\infty} N\left(f(xy) - f(x)f(y), t\varphi(x, y, 0)\right) = 1 \tag{3}$$

uniformly on X^2. Then $H(x) := N\text{-}\lim_{n\to\infty} 2^n f\left(\frac{x}{2^n}\right)$ exists for each $x \in X$ and defines a fuzzy algebra homomorphism $H : X \to Y$ such that if for some $\delta > 0, \alpha > 0$

$$N\left(2f\left(\frac{rx + ry}{2} + rz\right) - rf(x) - rf(y) - 2rf(z), \delta\varphi(x, y, z)\right) \geq \alpha \tag{4}$$

for all $x, y, z \in X$ and all $r \in \mathbb{R}$, then

$$N\left(f(x) - H(x), \delta\widetilde{\varphi}(x, x, x)\right) \geq \alpha \tag{5}$$

for all $x \in X$.

Furthermore, the fuzzy algebra homomorphism $H : X \to Y$ is a unique mapping such that

$$\lim_{t\to\infty} N(f(x) - H(x), t\widetilde{\varphi}(x, x, x)) = 1 \tag{6}$$

uniformly on X.

Proof. Let $r = 1$ in (2). For a given $\varepsilon > 0$, by (2), we can find some $t_0 > 0$ such that

$$N\left(2f\left(\frac{x+y}{2}+z\right) - f(x) - f(y) - 2f(z), t\varphi(x,y,z)\right) \geq 1-\varepsilon \tag{7}$$

for all $t \geq t_0$ and all $x, y, z \in X$. Letting $y = x, z = x$ in (7), we get

$$N(2f(2x) - 4f(x), t\varphi(x,x,x)) \geq 1-\epsilon \tag{8}$$

for all $x \in X$.

By induction on n, we will show that

$$N\left(f(x) - 2^n f\left(\frac{x}{2^n}\right), t\sum_{k=1}^{n} 2^{k-2}\varphi\left(\frac{x}{2^k}, \frac{x}{2^k}, \frac{x}{2^k}\right)\right) \geq 1-\varepsilon \tag{9}$$

for all $t \geq t_0$, all $x \in X$ and all $n \in \mathbb{N}$.

It follows from (8) that

$$N\left(f(x) - 2f\left(\frac{x}{2}\right), \frac{t}{2}\varphi\left(\frac{x}{2}, \frac{x}{2}, \frac{x}{2}\right)\right) \geq 1-\epsilon$$

for all $x \in X$.

Thus we get (9) for $n = 1$.

Assume that (9) holds for $n \in \mathbb{N}$. Then

$$\begin{aligned} &N\left(f(x) - 2^{n+1} f\left(\frac{x}{2^{n+1}}\right), t\sum_{k=1}^{n+1} 2^{k-2}\varphi\left(\frac{x}{2^k}, \frac{x}{2^k}, \frac{x}{2^k}\right)\right) \\ &\geq \min\left\{N\left(f(x) - 2^n f\left(\frac{x}{2^n}\right), t\sum_{k=1}^{n} 2^{k-2}\varphi\left(\frac{x}{2^k}, \frac{x}{2^k}, \frac{x}{2^k}\right)\right),\right. \\ &\qquad \left. N\left(2^n f\left(\frac{x}{2^n}\right) - 2^{n+1} f\left(\frac{x}{2^{n+1}}\right), 2^{n-1}t\varphi\left(\frac{x}{2^{n+1}}, \frac{x}{2^{n+1}}, \frac{x}{2^{n+1}}\right)\right)\right\} \\ &\geq \min\{1-\varepsilon, 1-\varepsilon\} = 1-\varepsilon. \end{aligned}$$

This completes the induction argument. Letting $t = t_0$ and replacing n and x by p and $\frac{x}{2^n}$ in (9), respectively, we get

$$N\left(2^n f\left(\frac{x}{2^n}\right) - 2^{n+p} f\left(\frac{x}{2^{n+p}}\right), 2^n t_0 \sum_{k=1}^{p} 2^{k-2}\varphi\left(\frac{x}{2^{n+k}}, \frac{x}{2^{n+k}}, \frac{x}{2^{n+k}}\right)\right) \geq 1-\varepsilon \tag{10}$$

for all integers $n \geq 0, p > 0$.

It follows from (1) and the equality

$$\sum_{k=1}^{p} 2^{n+k-2}\varphi\left(\frac{x}{2^{n+k}}, \frac{x}{2^{n+k}}, \frac{x}{2^{n+k}}\right) = \sum_{k=n+1}^{n+p} 2^{k-2}\varphi\left(\frac{x}{2^{k}}, \frac{x}{2^{k}}, \frac{x}{2^{k}}\right)$$

that for a given $\delta > 0$ there is an $n_0 \in \mathbb{N}$ such that

$$t_0 \sum_{k=n+1}^{n+p} 2^{k-2}\varphi\left(\frac{x}{2^{k}}, \frac{x}{2^{k}}, \frac{x}{2^{k}}\right) < \delta$$

for all $n \geq n_0$ and $p > 0$. Now we deduce from (10) that

$$\begin{aligned} & N\left(2^n f\left(\frac{x}{2^n}\right) - 2^{n+p} f\left(\frac{x}{2^{n+p}}\right), \delta\right) \\ & \geq N\left(2^n f\left(\frac{x}{2^n}\right) - 2^{n+p} f\left(\frac{x}{2^{n+p}}\right), 2^n t_0 \sum_{k=1}^{p} 2^{k-2}\varphi\left(\frac{x}{2^{n+k}}, \frac{x}{2^{n+k}}, \frac{x}{2^{n+k}}\right)\right) \\ & \geq 1 - \varepsilon \end{aligned}$$

for each $n \geq n_0$ and all $p > 0$. Thus the sequence $\left\{2^n f\left(\frac{x}{2^n}\right)\right\}$ is Cauchy in Y. Since Y is a fuzzy Banach space, the sequence $\left\{2^n f\left(\frac{x}{2^n}\right)\right\}$ converges to some $H(x) \in Y$. So we can define a mapping $H : X \to Y$ by $H(x) := N\text{-}\lim_{n\to\infty} 2^n f\left(\frac{x}{2^n}\right)$, namely, for each $t > 0$ and $x \in X$,

$$\lim_{n\to\infty} N\left(2^n f\left(\frac{x}{2^n}\right) - H(x), t\right) = 1.$$

Fix $t > 0$ and $0 < \varepsilon < 1$. Since $\lim_{n\to\infty} 2^n \varphi\left(\frac{x}{2^n}, \frac{y}{2^n}, \frac{z}{2^n}\right) = 0$, there is an $n_1 > n_0$ such that $2^n t_0 \varphi\left(\frac{x}{2^n}, \frac{y}{2^n}, \frac{z}{2^n}\right) < \frac{t}{5}$ for all $n \geq n_1$. Hence for each $k \geq n_1$, we have

$$\begin{aligned} & N\left(2H\left(\frac{rx+ry}{2} + rz\right) - rH(x) - rH(y) - 2rH(z), t\right) \\ & \geq \min\left\{N\left(2H\left(\frac{rx+ry}{2} + rz\right) - 2^{k+1} f\left(\frac{rx+ry}{2^{k+1}} + \frac{rz}{2^k}\right), \frac{t}{5}\right),\right. \\ & \quad N\left(rH(x) - 2^k r f\left(\frac{x}{2^k}\right), \frac{t}{5}\right), N\left(rH(y) - 2^k r f\left(\frac{y}{2^k}\right), \frac{t}{5}\right), \\ & \quad N\left(2rH(z) - 2^{k+1} r f\left(\frac{z}{2^k}\right), \frac{t}{5}\right), \\ & \left. N\left(2^{k+1} f\left(\frac{rx+ry}{2^{k+1}} + \frac{rz}{2^k}\right) - 2^k r f\left(\frac{x}{2^k}\right) - 2^k r f\left(\frac{y}{2^k}\right) - 2^{k+1} r f\left(\frac{z}{2^k}\right), \frac{t}{5}\right)\right\}. \end{aligned}$$

The first four terms on the right-hand side of the above inequality tend to 1 as $k \to \infty$, and the last term is greater than

$$N\left(2^{k+1} f\left(\frac{rx+ry}{2^{k+1}}+\frac{rz}{2^k}\right)-2^k rf\left(\frac{x}{2^k}\right)-2^k rf\left(\frac{y}{2^k}\right)-2^{k+1} rf\left(\frac{z}{2^k}\right),\right.$$
$$\left. 2^k t_0 \varphi\left(\frac{x}{2^k},\frac{y}{2^k},\frac{z}{2^k}\right)\right),$$

which is greater than or equal to $1-\varepsilon$ by (2). Thus

$$N\left(2H\left(\frac{rx+ry}{2}+rz\right)-rH(x)-rH(y)-2rH(z),t\right) \geq 1-\varepsilon$$

for all $t>0$ and each $r \in \mathbb{R}$. Since

$$N\left(2H\left(\frac{rx+ry}{2}+rz\right)-rH(x)-rH(y)-2rH(z),t\right) = 1$$

for all $t>0$, by (N_2),

$$2H\left(\frac{rx+ry}{2}+rz\right)-rH(x)-rH(y)-2rH(z)=0 \tag{11}$$

for all $x,y,z \in X$ and each $r \in \mathbb{R}$. Hence the mapping $H : X \to Y$ is Cauchy-Jensen additive.

It follows from (11) that $H(rx) = rH(x)$ for all $r \in \mathbb{R}$ and all $x \in X$.

Similarly, it follows from (3) that $H(xy) = H(x)H(y)$ for all $x, y \in X$. □

Similarly, we can obtain the following. We will omit the proof.

THEOREM 2.2. *Let $\varphi : X^3 \to [0,\infty)$ be a function such that*

$$\widetilde{\varphi}(x,y,z) := \sum_{n=0}^{\infty} \frac{1}{2^n}\varphi(2^n x, 2^n y, 2^n z) < \infty$$

for all $x,y,z \in X$. Let $f : X \to Y$ be a mapping satisfying (2) and (3). Then $H(x) := N\text{-}\lim_{n\to\infty} \frac{1}{2^n} f(2^n x)$ exists for each $x \in X$ and defines a fuzzy algebra homomorphism $H : X \to Y$ such that if for some $\delta > 0, \alpha > 0$

$$N\left(2f\left(\frac{rx+ry}{2}+rz\right)-rf(x)-rf(y)-2rf(z),\delta\varphi(x,y,z)\right) \geq \alpha$$

for all $x,y,z \in X$ and all $r \in \mathbb{R}$, then

$$N\left(f(x)-H(x),\delta\widetilde{\varphi}(x,x,x)\right) \geq \alpha$$

for all $x \in X$.

Furthermore, the fuzzy algebra homomorphism $H : X \to Y$ *is a unique mapping such that*

$$\lim_{t\to\infty} N\left(f(x) - H(x), t\widetilde{\varphi}\left(x, x, x\right)\right) = 1$$

uniformly on X.

3. Hyers-Ulam stability of derivations on fuzzy Banach algebras

In this section, using the direct method, we prove the Hyers-Ulam stability of derivations on fuzzy Banach algebras associated with the Cauchy-Jensen functional equation.

THEOREM 3.1. *Let* $\varphi : Y^3 \to [0, \infty)$ *be a function such that*

$$\widetilde{\varphi}(x, y, z) := \sum_{j=1}^{\infty} 2^j \varphi\left(\frac{x}{2^j}, \frac{y}{2^j}, \frac{z}{2^j}\right) < \infty$$

for all $x, y, z \in Y$. *Let* $f : Y \to Y$ *be a mapping satisfying* $f(0) = 0$ *and*

$$\lim_{t\to\infty} N\left(2f\left(\frac{rx + ry}{2} + rz\right) - rf(x) - rf(y) - 2rf(z), t\varphi(x, y, z)\right) = 1 \tag{12}$$

uniformly on Y^3 *for each* $r \in \mathbb{R}$, *and*

$$\lim_{t\to\infty} N\left(f(xy) - f(x)y - xf(y), t\varphi(x, y, 0)\right) = 1 \tag{13}$$

uniformly on Y^2. *Then* $D(x) := N\text{-}\lim_{n\to\infty} 2^n f\left(\frac{x}{2^n}\right)$ *exists for each* $x \in Y$ *and defines a fuzzy derivation* $D : Y \to Y$ *such that if for some* $\delta > 0, \alpha > 0$

$$N\left(2f\left(\frac{rx + ry}{2} + rz\right) - rf(x) - rf(y) - 2rf(z), \delta\varphi(x, y, z)\right) \geq \alpha$$

for all $x, y, z \in Y$ *and all* $r \in \mathbb{R}$, *then*

$$N\left(f(x) - D(x), \delta\widetilde{\varphi}(x, x, x)\right) \geq \alpha$$

for all $x \in Y$.

Furthermore, the fuzzy derivation $D : Y \to Y$ is a unique mapping such that

$$\lim_{t\to\infty} N(f(x) - D(x), t\widetilde{\varphi}(x,x,x)) = 1 \tag{14}$$

uniformly on Y.

Proof. By the same reasoning as in the proof of Theorem 2.1, one can show that there exists a unique $\mathbb{R}$-linear mapping $D : Y \to Y$ satisfying (14). The $\mathbb{R}$-linear mapping $D : Y \to Y$ is defined by $D(x) := N$-$\lim_{n\to\infty} 2^n f\left(\frac{x}{2^n}\right)$ for all $x \in Y$.

By a similar method to the proof of Theorem 2.1, one can show that $D(xy) = D(x)y + xD(y)$ for all $x, y \in Y$, as desired. □

Similarly, we can obtain the following. We will omit the proof.

THEOREM 3.2. *Let $\varphi : Y^3 \to [0,\infty)$ be a function such that*

$$\widetilde{\varphi}(x,y,z) := \sum_{n=0}^{\infty} \frac{1}{2^n}\varphi\left(2^n x, 2^n y, 2^n z\right) < \infty$$

for all $x, y, z \in Y$. Let $f : Y \to Y$ be a mapping satisfying (12) *and* (13)*. Then $D(x) := N$-$\lim_{n\to\infty} \frac{1}{2^n} f(2^n x)$ exists for each $x \in Y$ and defines a fuzzy derivation $D : Y \to Y$ such that if for some $\delta > 0, \alpha > 0$*

$$N\left(2f\left(\frac{rx+ry}{2} + rz\right) - rf(x) - rf(y) - 2rf(z), \delta\varphi(x,y,z)\right) \geq \alpha$$

for all $x, y, z \in Y$ and all $r \in \mathbb{R}$, then

$$N\left(f(x) - D(x), \delta\widetilde{\varphi}\left(x,x,x\right)\right) \geq \alpha$$

for all $x \in Y$.

Furthermore, the fuzzy derivation $D : Y \to Y$ is a unique mapping such that

$$\lim_{t\to\infty} N\left(f(x) - D(x), t\widetilde{\varphi}\left(x,x,x\right)\right) = 1$$

uniformly on Y.

References

[1] T. Aoki, *On the stability of the linear transformation in Banach spaces*, J. Math. Soc. Japan **2** (1950), 64–66.

[2] T. Bag and S.K. Samanta, *Finite dimensional fuzzy normed linear spaces*, J. Fuzzy Math. **11** (2003), 687–705.

[3] T. Bag and S.K. Samanta, *Fuzzy bounded linear operators*, Fuzzy Sets and Systems **151** (2005), 513–547.
[4] L. Cădariu and V. Radu, *Fixed points and the stability of Jensen's functional equation*, J. Inequal. Pure Appl. Math. **4**, no. 1, Art. ID 4 (2003).
[5] L. Cădariu and V. Radu, *On the stability of the Cauchy functional equation: a fixed point approach*, Grazer Math. Ber. **346** (2004), 43–52.
[6] L. Cădariu and V. Radu, *Fixed point methods for the generalized stability of functional equations in a single variable*, Fixed Point Theory and Applications **2008**, Art. ID 749392 (2008).
[7] S.C. Cheng and J.M. Mordeson, *Fuzzy linear operators and fuzzy normed linear spaces*, Bull. Calcutta Math. Soc. **86** (1994), 429–436.
[8] Y. Cho, C. Park and R. Saadati, *Functional inequalities in non-Archimedean Banach spaces*, Appl. Math. Letters **23** (2010), 1238–1242.
[9] P.W. Cholewa, *Remarks on the stability of functional equations*, Aequationes Math. **27** (1984), 76–86.
[10] S. Czerwik, *On the stability of the quadratic mapping in normed spaces*, Abh. Math. Sem. Univ. Hamburg **62** (1992), 59–64.
[11] S. Czerwik, *The stability of the quadratic functional equation.* in: Stability of mappings of Hyers-Ulam type, (ed. Th.M. Rassias and J.Tabor), Hadronic Press, Palm Harbor, Florida, 1994, 81-91.
[12] P. Czerwik, *Functional Equations and Inequalities in Several Variables*, World Scientific Publishing Company, New Jersey, Hong Kong, Singapore and London, 2002.
[13] J. Diaz and B. Margolis, *A fixed point theorem of the alternative for contractions on a generalized complete metric space*, Bull. Amer. Math. Soc. **74** (1968), 305–309.
[14] M. Eshaghi Gordji, *A characterization of* (σ, τ)*-derivations on von Neumann algebras*, Politehn. Univ. Bucharest Sci. Bull. Ser. A–Appl. Math. Phys. **73** (2011), No. 1, 111–116.
[15] M. Eshaghi Gordji, A. Bodaghi and C. Park, *A fixed point approach to the stability of double Jordan centralizers and Jordan multipliers on Banach algebras*, Politehn. Univ. Bucharest Sci. Bull. Ser. A–Appl. Math. Phys. **73** (2011), No. 2, 65–74.
[16] M. Eshaghi Gordji, H. Khodaei and R. Khodabakhsh, *General quartic-cubic-quadratic functional equation in non-Archimedean normed spaces*, Politehn. Univ. Bucharest Sci. Bull. Ser. A–Appl. Math. Phys. **72** (2010), No. 3, 69–84.
[17] M. Eshaghi Gordji and M.B. Savadkouhi, *Approximation of generalized homomorphisms in quasi-Banach algebras*, An. Stiint. Univ. Ovidius Constanta Ser. Mat. **17** (2009), No. 2, 203–213.
[18] C. Felbin, *Finite dimensional fuzzy normed linear spaces*, Fuzzy Sets and Systems **48** (1992), 239–248.
[19] P. Găvruta, *A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings*, J. Math. Anal. Appl. **184** (1994), 431–436.
[20] D.H. Hyers, *On the stability of the linear functional equation*, Proc. Natl. Acad. Sci. USA **27** (1941), 222–224.

[21] D.H. Hyers, G. Isac and Th.M. Rassias, *Stability of Functional Equations in Several Variables*, Birkhäuser, Basel, 1998.

[22] G. Isac and Th.M. Rassias, *Stability of ψ-additive mappings: Appications to nonlinear analysis*, Internat. J. Math. Math. Sci. **19** (1996), 219–228.

[23] S. Jung, *Hyers-Ulam-Rassias Stability of Functional Equations in Mathematical Analysis*, Hadronic Press lnc., Palm Harbor, Florida, 2001.

[24] A.K. Katsaras, *Fuzzy topological vector spaces II*, Fuzzy Sets and Systems **12** (1984), 143–154.

[25] I. Kramosil and J. Michalek, *Fuzzy metric and statistical metric spaces*, Kybernetica **11** (1975), 326–334.

[26] S.V. Krishna and K.K.M. Sarma, *Separation of fuzzy normed linear spaces*, Fuzzy Sets and Systems **63** (1994), 207–217.

[27] D. Miheţ and V. Radu, *On the stability of the additive Cauchy functional equation in random normed spaces*, J. Math. Anal. Appl. **343** (2008), 567–572.

[28] M. Mirzavaziri and M.S. Moslehian, *A fixed point approach to stability of a quadratic equation*, Bull. Braz. Math. Soc. **37** (2006), 361–376.

[29] A.K. Mirmostafaee, M. Mirzavaziri and M.S. Moslehian, *Fuzzy stability of the Jensen functional equation*, Fuzzy Sets and Systems **159** (2008), 730–738.

[30] A.K. Mirmostafaee and M.S. Moslehian, *Fuzzy versions of Hyers-Ulam-Rassias theorem*, Fuzzy Sets and Systems **159** (2008), 720–729.

[31] A.K. Mirmostafaee and M.S. Moslehian, *Fuzzy approximately cubic mappings*, Inform. Sci. **178** (2008), 3791–3798.

[32] C. Park, *Fixed points and Hyers-Ulam-Rassias stability of Cauchy-Jensen functional equations in Banach algebras*, Fixed Point Theory and Applications **2007**, Art. ID 50175 (2007).

[33] C. Park, *Generalized Hyers-Ulam-Rassias stability of quadratic functional equations: a fixed point approach*, Fixed Point Theory and Applications **2008**, Art. ID 493751 (2008).

[34] V. Radu, *The fixed point alternative and the stability of functional equations*, Fixed Point Theory **4** (2003), 91–96.

[35] Th.M. Rassias, *On the stability of the linear mapping in Banach spaces*, Proc. Amer. Math. Soc. **72** (1978), 297–300.

[36] R. Saadati and C. Park, *Non-Archimedean $\mathcal{L}$-fuzzy normed spaces and stability of functional equations*, Computers Math. Appl. **60** (2010), 2488–2496.

[37] F. Skof, *Proprietà locali e approssimazione di operatori*, Rend. Sem. Mat. Fis. Milano **53** (1983), 113–129.

[38] S. M. Ulam, *A Collection of the Mathematical Problems*, Interscience Publ. New York, 1960.

[39] J.Z. Xiao and X.H. Zhu, *Fuzzy normed spaces of operators and its completeness*, Fuzzy Sets and Systems **133** (2003), 389–399.

Jung Rye Lee
Department of Mathematics
Daejin University
Kyeonggi 487-711, Korea
E-mail: jrlee@daejin.ac.kr