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FUZZY ALMOST STRONGLY (r, s)-SEMIOPEN AND SEMICLOSED MAPPINGS

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Abstract. In this paper, we introduce the concepts of fuzzy almost strongly (r, s)-semiopen and semiclosed mappings on intuitionistic fuzzy topological spaces in Šostak's sense, and investigate some of their characteristic properties.

1. Introduction

The concept of fuzzy set was introduced by Zadeh [13]. Chang [3] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which, developed by Šostak [12], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay and his colleagues [4], and by Ramadan [11].

As a generalization of fuzzy sets, Atanassov [1] introduced the concept of intuitionistic fuzzy sets, and Çoker [5] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and Demirci [6] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth topological spaces and intuitionistic fuzzy topological spaces. Shi-Zhong Bai [2] introduced the concepts of fuzzy almost

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strongly semiopen and semiclosed mappings on Chang's fuzzy topological spaces.

In this paper, we introduce the concepts of fuzzy almost strongly (r,s)-semiopen and semiclosed mappings on intuitionistic fuzzy topological spaces in Šostak's sense, and investigate some of their characteristic properties.

2. Preliminaries

For the nonstandard definitions and notations we refer to [7–10].

Let $I(X)$ be a family of all intuitionistic fuzzy sets in X and let $I\otimes I$ be the set of the pair (r,s) such that $r,s\in I$ and $r+s\le 1$.

Definition 2.1. ([6]) Let X be a nonempty set. An *intuitionistic fuzzy topology in Šostak's sense*(SoIFT for short) $\mathcal{T}=(\mathcal{T}_1,\mathcal{T}_2)$ on X is a mapping $\mathcal{T}:I(X)\to I\otimes I$ which satisfies the following properties:

(1) $\mathcal{T}_1(\underline{0})=\mathcal{T}_1(\underline{1})=1$ and $\mathcal{T}_2(\underline{0})=\mathcal{T}_2(\underline{1})=0$.
(2) $\mathcal{T}_1(A\cap B)\ge\mathcal{T}_1(A)\wedge\mathcal{T}_1(B)$ and $\mathcal{T}_2(A\cap B)\le\mathcal{T}_2(A)\vee\mathcal{T}_2(B)$.
(3) $\mathcal{T}_1(\bigcup A_i)\ge\bigwedge\mathcal{T}_1(A_i)$ and $\mathcal{T}_2(\bigcup A_i)\le\bigvee\mathcal{T}_2(A_i)$.

The $(X,\mathcal{T})=(X,\mathcal{T}_1,\mathcal{T}_2)$ is said to be an *intuitionistic fuzzy topological space in Šostak's sense*(SoIFTS for short). Also, we call $\mathcal{T}_1(A)$ a *gradation of openness* of A and $\mathcal{T}_2(A)$ a *gradation of nonopenness* of A.

Definition 2.2. ([7, 8, 10]) Let A be an intuitionistic fuzzy set in a SoIFTS $(X,\mathcal{T}_1,\mathcal{T}_2)$ and $(r,s)\in I\otimes I$. Then A is said to be

(1) *fuzzy (r,s)-semiopen* if $\mathrm{cl}(\mathrm{int}(A,r,s),r,s)\supseteq A$,
(2) *fuzzy (r,s)-semiclosed* if $\mathrm{int}(\mathrm{cl}(A,r,s),r,s)\subseteq A$,
(3) *fuzzy (r,s)-regular open* if $\mathrm{int}(\mathrm{cl}(A,r,s),r,s)=A$,
(4) *fuzzy (r,s)-regular closed* if $\mathrm{cl}(\mathrm{int}(A,r,s),r,s)=A$,
(5) *fuzzy strongly (r,s)-semiopen* if $A\subseteq\mathrm{int}(\mathrm{cl}(\mathrm{int}(A,r,s),r,s),r,s)$,
(6) *fuzzy strongly (r,s)-semiclosed* if $A\supseteq\mathrm{cl}(\mathrm{int}(\mathrm{cl}(A,r,s),r,s),r,s)$.

Definition 2.3. ([10]) Let $(X,\mathcal{T}_1,\mathcal{T}_2)$ be a SoIFTS. For each $(r,s)\in I\otimes I$ and for each $A\in I(X)$, the *fuzzy strongly (r,s)-semiinterior* is defined by

$$\mathrm{ssint}(A,r,s)=\bigcup\{B\in I(X)\mid B\subseteq A,\ B\text{ is fuzzy strongly }(r,s)\text{-semiopen}\}$$

and the *fuzzy strongly* (r, s)*-semiclosure* is defined by

$$\text{sscl}(A, r, s) = \bigcap\{B \in I(X) \mid A \subseteq B,\ B \text{ is fuzzy strongly } (r, s)\text{-semiclosed}\}.$$

THEOREM 2.4. ([8]) *(1) The fuzzy* (r, s)*-closure of a fuzzy* (r, s)*-open set is fuzzy* (r, s)*-regular closed for each* $(r, s) \in I \otimes I$.
(2) The fuzzy (r, s)*-interior of a fuzzy* (r, s)*-closed set is fuzzy* (r, s)*-regular open for each* $(r, s) \in I \otimes I$.

DEFINITION 2.5. ([2]) Let $f : (X_1, \delta_1) \to (X_2, \delta_2)$ be a mapping from a fuzzy topological space X_1 to another fuzzy topological space X_2. Then f is called

(1) a *fuzzy almost strongly semiopen* mapping if $f(A)$ is a fuzzy strongly semiopen set of X_2 for each fuzzy regular open set A of X_1,
(2) a *fuzzy almost strongly semiclosed* mapping if $f(A)$ is a fuzzy strongly semiclosed set of X_2 for each fuzzy regular closed set A of X_1.

3. Fuzzy almost strongly (r, s)-semiopen and semiclosed mappings

Now, we define the notions of fuzzy almost strongly (r, s)-semiopen and semiclosed mappings on intuitionistic fuzzy topological spaces in Šostak's sense, and then we investigate some of their properties.

DEFINITION 3.1. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then f is called

(1) a *fuzzy almost strongly* (r, s)*-semiopen* mapping if $f(A)$ is a fuzzy strongly (r, s)-semiopen set in Y for each fuzzy (r, s)-regular open set A in X,
(2) a *fuzzy almost strongly* (r, s)*-semiclosed* mapping if $f(A)$ is a fuzzy strongly (r, s)-semiclosed set in Y for each fuzzy (r, s)-regular closed set A in X.

THEOREM 3.2. *Let* $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ *be a mapping from a SoIFTS* X *to a SoIFTS* Y *and* $(r, s) \in I \otimes I$. *Then the following statements are equivalent:*

(1) f *is fuzzy almost strongly* (r, s)*-semiopen.*

(2) *For each fuzzy (r, s)-open set A in X,*

$$f(A) \subseteq ssint(f(int(cl(A, r, s), r, s)), r, s).$$

(3) *For each fuzzy (r, s)-semiclosed set A in X,*

$$f(int(A, r, s)) \subseteq ssint(f(A), r, s).$$

(4) *For each intuitionistic fuzzy set B in Y and each fuzzy (r, s)-regular closed set A in X with $f^{-1}(B) \subseteq A$, there is a fuzzy strongly (r, s)-semiclosed set C in Y such that $B \subseteq C$ and $f^{-1}(C) \subseteq A$.*

Proof. (1) $\Rightarrow$ (2) Let A be a fuzzy (r, s)-open set in X. By Theorem 2.4, $\mathrm{int}(\mathrm{cl}(A, r, s), r, s)$ is fuzzy (r, s)-regular open in X. Since f is a fuzzy almost strongly (r, s)-semiopen mapping, $f(\mathrm{int}(\mathrm{cl}(A, r, s), r, s))$ is fuzzy strongly (r, s)-semiopen in Y. Hence we have

$$\begin{aligned} f(A) = f(\mathrm{int}(A, r, s)) &\subseteq f(\mathrm{int}(\mathrm{cl}(A, r, s), r, s)) \\ &= \mathrm{ssint}(f(\mathrm{int}(\mathrm{cl}(A, r, s), r, s)), r, s). \end{aligned}$$

(2) $\Rightarrow$ (3) Let A be a fuzzy (r, s)-semiclosed set in X. Then $\mathrm{int}(A, r, s)$ is fuzzy (r, s)-open in X. Thus by (2), we have

$$\begin{aligned} f(\mathrm{int}(A, r, s)) &\subseteq \mathrm{ssint}(f(\mathrm{int}(\mathrm{cl}(\mathrm{int}(A, r, s), r, s), r, s)), r, s) \\ &\subseteq \mathrm{ssint}(f(\mathrm{int}(\mathrm{cl}(A, r, s), r, s)), r, s) \\ &\subseteq \mathrm{ssint}(f(A), r, s). \end{aligned}$$

(3) $\Rightarrow$ (1) Let A be a fuzzy (r, s)-regular open set in X. Then A is fuzzy (r, s)-open and also fuzzy (r, s)-semiclosed in X. Hence by (3), we obtain

$$f(A) = f(\mathrm{int}(A, r, s)) \subseteq \mathrm{ssint}(f(A), r, s) \subseteq f(A).$$

Thus $f(A) = \mathrm{ssint}(f(A), r, s)$, which is a fuzzy strongly (r, s)-semiopen set in Y. Therefore f is fuzzy almost strongly (r, s)-semiopen.

(1) $\Rightarrow$ (4) Let B be any intuitionistic fuzzy set in Y and A a fuzzy (r, s)-regular closed set in X with $f^{-1}(B) \subseteq A$. Then $A^c \subseteq f^{-1}(B^c)$, and hence $f(A^c) \subseteq f(f^{-1}(B^c)) \subseteq B^c$. Since f is fuzzy almost strongly (r, s)-semiopen and A^c is fuzzy (r, s)-regular open, we have $f(A^c) \subseteq \mathrm{ssint}(B^c, r, s)$. Thus $A^c \subseteq f^{-1}(f(A^c)) \subseteq f^{-1}(\mathrm{ssint}(B^c, r, s))$. Hence $A \supseteq f^{-1}(\mathrm{sscl}(B, r, s))$. Let $C = \mathrm{sscl}(B, r, s)$. Then C is fuzzy strongly (r, s)-semiclosed such that $B \subseteq C$ and $f^{-1}(C) \subseteq A$.

(4) $\Rightarrow$ (1) Let A be a fuzzy (r,s)-regular open set in X. Then A^c is fuzzy (r,s)-regular closed. Note that $A^c \supseteq (f^{-1}(f(A)))^c = f^{-1}(f(A)^c)$. According to the assumption, there is a fuzzy strongly (r,s)-semiclosed set B in Y such that $f(A)^c \subseteq B$ and $f^{-1}(B) \subseteq A^c$. From $f(A)^c \subseteq B$ we have $\mathrm{sscl}(f(A)^c, r, s) \subseteq B$, and hence $B^c \subseteq \mathrm{sscl}(f(A)^c, r, s)^c = \mathrm{ssint}(f(A), r, s)$. Since $f^{-1}(B) \subseteq A^c$, we obtain $f^{-1}(B^c) \supseteq A$, and thus $B^c \supseteq f(f^{-1}(B^c)) \supseteq f(A)$. Hence $f(A) = \mathrm{ssint}(f(A), r, s)$. Thus $f(A)$ is a fuzzy strongly (r,s)-semiopen set in Y. Therefore f is almost strongly (r,s)-semiopen. □

Theorem 3.3. *Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then f is fuzzy almost strongly (r,s)-semiopen if and only if $f(\mathrm{int}(A,r,s)) \subseteq \mathrm{ssint}(f(A),r,s)$ for each fuzzy (r,s)-semiclosed set A in X.*

Proof. ($\Rightarrow$) Let f be fuzzy almost strongly (r,s)-semiopen and A a fuzzy (r,s)-semiclosed set in X. Then

$$\mathrm{int}(A,r,s) \subseteq \mathrm{int}(\mathrm{cl}(A,r,s),r,s) \subseteq A.$$

By Theorem 2.4, $\mathrm{int}(\mathrm{cl}(A,r,s),r,s)$ is a fuzzy (r,s)-regular open set in X. Since f is fuzzy almost strongly (r,s)-semiopen, $f(\mathrm{int}(\mathrm{cl}(A,r,s),r,s))$ is fuzzy strongly (r,s)-semiopen in Y. Hence we obtain

$$\begin{aligned} f(\mathrm{int}(A,r,s)) &\subseteq f(\mathrm{int}(\mathrm{cl}(A,r,s),r,s)) \\ &= \mathrm{ssint}(f(\mathrm{int}(\mathrm{cl}(A,r,s),r,s)),r,s) \\ &\subseteq \mathrm{ssint}(f(A),r,s). \end{aligned}$$

Conversely, let A be a fuzzy (r,s)-regular open set in X. Then A is fuzzy (r,s)-open in X, and hence $\mathrm{int}(A,r,s) = A$. Since $\mathrm{int}(\mathrm{cl}(A,r,s),r,s) = A$, A is fuzzy (r,s)-semiclosed in X. Hence

$$f(A) = f(\mathrm{int}(A,r,s)) \subseteq \mathrm{ssint}(f(A),r,s) \subseteq\subseteq f(A).$$

Thus $f(A) = \mathrm{ssint}(f(A),r,s)$, which is a fuzzy strongly (r,s)-semiopen set in Y. Therefore f is a fuzzy almost strongly (r,s)-semiopen mapping. □

Theorem 3.4. *Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then the following statements are equivalent:*

(1) *f is fuzzy almost strongly (r,s)-semiclosed.*

(2) *For each fuzzy (r,s)-closed set A in X,*

$$sscl(f(cl(int(A,r,s),r,s)),r,s) \subseteq f(A).$$

(3) *For each fuzzy (r,s)-semiopen set B in Y,*

$$sscl(f(A),r,s) \subseteq f(cl(A,r,s)).$$

Proof. (1) $\Rightarrow$ (2) Let A be a fuzzy (r,s)-closed set in X. By Theorem 2.4, $\mathrm{cl}(\mathrm{int}(A,r,s),r,s)$ is fuzzy (r,s)-regular closed in X. Since f is fuzzy almost strongly (r,s)-semiclosed, $f(\mathrm{cl}(\mathrm{int}(A,r,s),r,s))$ is fuzzy strongly (r,s)-semiclosed in Y. Hence we have

$$\begin{aligned} \mathrm{sscl}(f(\mathrm{cl}(\mathrm{int}(A,r,s),r,s)),r,s) &= f(\mathrm{cl}(\mathrm{int}(A,r,s),r,s)) \\ &\subseteq f(\mathrm{cl}(A,r,s)) = f(A). \end{aligned}$$

(2) $\Rightarrow$ (3) Let A be a fuzzy (r,s)-semiopen set in X. Then $\mathrm{cl}(A,r,s)$ is fuzzy (r,s)-closed in X. By (2), we have

$$\begin{aligned} \mathrm{sscl}(f(A),r,s) &\subseteq \mathrm{sscl}(f(\mathrm{cl}(A,r,s)),r,s) \\ &\subseteq \mathrm{sscl}(f(\mathrm{cl}(\mathrm{int}(\mathrm{cl}(A,r,s),r,s),r,s)),r,s) \\ &\subseteq f(\mathrm{cl}(A,r,s)). \end{aligned}$$

(3) $\Rightarrow$ (1) Let A be a fuzzy (r,s)-regular closed set in X. Then A is fuzzy (r,s)-closed and fuzzy (r,s)-semiopen in X. By (3), we obtain

$$f(A) \subseteq \mathrm{sscl}(f(A),r,s) \subseteq f(\mathrm{cl}(A,r,s)) = f(A).$$

Thus we have $f(A) = \mathrm{sscl}(f(A),r,s)$, which is a fuzzy strongly (r,s)-semiclosed set in Y. Hence f is fuzzy almost strongly (r,s)-semiclosed. □

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