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THE JEU DE TAQUIN ON THE SHIFTED RIM HOOK TABLEAUX

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Abstract. The Schensted algorithm first described by Robinson [5] is a remarkable combinatorial correspondence associated with the theory of symmetric functions. Schützenberger's jeu de taquin[10] can be used to give alternative descriptions of both P- and Q-tableaux of the Schensted algorithm as well as the ordinary and dual Knuth relations. In this paper we describe the jeu de taquin on shifted rim hook tableaux using the switching rule, which shows that the sum of the weights of the shifted rim hook tableaux of a given shape and content does not depend on the order of the content if content parts are all odd.

1. Introduction

There is a remarkable combinatorial correspondence associated with the theory of symmetric functions, called the Schensted algorithm.

Theorem 1.1. (**Schensted algorithm**) *Let S_n be the symmetric group of degree n. Then there is a bijection*

$$\pi \mapsto (P, Q)$$

between permutations π of S_n and the set of all pairs $(P_\lambda\ ,\ Q_\lambda)$ of standard Young tableaux of the same shape λ, where $\lambda \vdash n$.

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It was first described in 1938 by Robinson [5], in a paper dealing with an attempt to prove the correctness of the Littlewood-Richardson rule. Schensted algorithm was rediscovered independently by Schensted [7] in 1961, whose main objective was counting permutations with given lengths of their longest increasing and decreasing subsequences. Schensted correspondence about increasing and decreasing subsequences is extended by C. Greene [3], to give a direct interpretation of the shape of the standard Young tableaux corresponding to a permutation. Knuth [4] gave a generalization of the Schensted algoritm, where standard Young tableaux are replaced by column strict tableaux, and permutations are replaced by multi-permutations. And he described conditions for two permutation to have the same P-tableaux under Schensted algorithm. In [11] Viennot gave a geometric interpretation for Schensted algorithm.

The combinatorial significance of Schensted algorithm was indicated by Schützenberger [10], who introduced the evacuation algorithm using the jeu de taquin. The jeu de taquin can be used to give alternative descriptions of both P- and Q-tableaux of the Schensted algorithm as well as the ordinary and dual Knuth relations.

After Schützenberger introduced the jeu de taquin on standard Young tableaux, various analogs of the jeu de taquin came: versions for rim hook tableaux [9] and shifted tableaux[6].

In this paper we describe the jeu de taquin on shifted rim hook tableaux using the switching rule, which shows that the sum of the weights of the shifted rim hook tableaux of a given shape and content does not depend on the order of the content if content parts are all odd. In section 2, we outline the definitions and notation used in this paper. In Section 3, we prove the switching rule for the shifted rim hook tableaux. The jeu de taquin on shifted rim hook tableaux is given in Section 4.

2. Definitions

We use standard notation $\mathbb{P}$ for the set of all positive integers.

Let $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_\ell)$ be a partition of the nonnegative integer n, denoted $\lambda \vdash n$ or $|\lambda| = n$, so λ is a weakly decreasing sequence of positive integers summing to n. We say each term λ_i is a *part* of λ and n is the *weight* of λ. The number of nonzero parts is called the *length* of λ and is written $\ell = \ell(\lambda)$. Let $\mathcal{P}_n$ be the set of all partitions of n and $\mathcal{P}$ be the set of all partitions. We also denote

$$DP = \{\mu \in \mathcal{P} \mid \mu \text{ has all distinct parts}\},$$
$$DP_n = \{\mu \in \mathcal{P}_n \mid \mu \text{ has all distinct parts}\}.$$

We sometimes abbreviate the partition λ with the notation $1^{j_1}2^{j_2}\ldots$, where j_i is the number of parts of size i. Sizes which do not appear are omitted and if $j_i = 1$, then it is not written. Thus, a partition $(5, 3, 2, 2, 2, 1) \vdash 15$ can be written 12^335.

For each $\lambda \in DP$, a *shifted diagram* D'_λ of shape λ is defined by

$$D'_\lambda = \{(i, j) \in \mathbb{P}^2 \mid i \le j \le \lambda_j + i - 1, 1 \le i \le \ell(\lambda)\}.$$

And for $\lambda, \mu \in DP$ with $D'_\mu \subseteq D'_\lambda$, a *shifted skew diagram* $D'_{\lambda/\mu}$ is defined as the set-theoretic difference $D'_\lambda \setminus D'_\mu$. Figure 2.1 and Figure 2.2 show D'_λ and $D'_{\lambda/\mu}$ respectively when $\lambda = (9, 7, 4, 2)$ and $\mu = (5, 3)$.

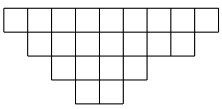

Figure 2.1

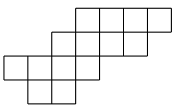

Figure 2.2

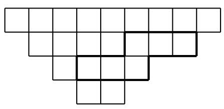

Figure 2.3

A shifted skew diagram θ is called a *single rim hook* if θ is connected and contains no 2×2 block of cells. If θ is a single rim hook, then its *head* is the upper rightmost cell in θ and its *tail* is the lower leftmost cell in θ. See Figure 2.3.

A *double rim hook* is a shifted skew diagram θ formed by the union of two single rim hooks both of whose tails are on the main diagonal. If θ is a double rim hook, we denote by $\mathcal{A}[\theta]$ (resp., $\alpha_1[\theta]$) the set of diagonals of length two (resp., one). Also let $\beta_1[\theta]$ (resp., $\gamma_1[\theta]$) be a single rim hook in θ which starts on the upper (resp., lower) of the two main diagonal cells and ends at the head of $\alpha_1[\theta]$. The tail of $\beta_1[\theta]$ (resp., $\gamma_1[\theta]$) is called the *first tail* (resp., *second tail*) of θ and the head of $\beta_1[\theta]$ or $\gamma_1[\theta]$ (resp., $\gamma_2[\theta], \beta_2[\theta]$, where $\beta_2[\theta] = \theta \setminus \beta_1[\theta]$ and $\gamma_2[\theta] = \theta \setminus \gamma_1[\theta]$) is called the *first head* (resp., *second head, third head*) of θ. Hence we have the following descriptions for a double rim hook θ:

$$\begin{aligned}\theta &= \mathcal{A}[\theta] \cup \alpha_1[\theta]\\ &= \beta_1[\theta] \cup \beta_2[\theta]\\ &= \gamma_1[\theta] \cup \gamma_2[\theta].\end{aligned}$$

A double rim hook is illustrated in Figure 2.4. We write $\mathcal{A}, \alpha_1$, etc. for $\mathcal{A}[\theta], \alpha_1[\theta]$, etc. when there is no confusion.

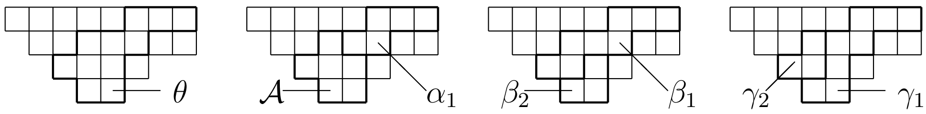

Figure 2.4

We will use the term *rim hook* to mean a single rim hook or a double rim hook.

A *shifted rim hook tableau* of shape $\lambda \in DP$ and content $\rho = (\rho_1, \ldots, \rho_m)$ is defined recursively. If $m = 1$, a rim hook with all 1's and shape λ is a shifted rim hook tableau. Suppose P of shape λ has content $\rho = (\rho_1, \rho_2, \ldots, \rho_m)$ and the cells containing the m's form a rim hook inside λ. If the removal of the m's leaves a shifted rim hook tableau, then P is a shifted rim hook tableau. We define a *shifted skew rim hook tableau* in a similar way. If P is a shifted rim hook tableau, we write $\kappa_P\langle r\rangle$ (or just $\kappa\langle r\rangle$) for a rim hook of P containing r.

If θ is a single rim hook then the *rank* $r(\theta)$ is one less than the number of rows it occupies and the *weight* $w(\theta) = (-1)^{r(\theta)}$; if θ is a double rim hook then the *rank* $r(\theta)$ is $|\mathcal{A}[\theta]|/2 + r(\alpha_1[\theta])$ and the *weight* $w(\theta)$ is $2(-1)^{r(\theta)}$.

The *weight* of a shifted rim hook tableau P, $w(P)$, is the product of the weights of its rim hooks. The weight of a shifted skew rim hook tableau is defined in a similar way.

1	1	1	1	2	2
	1	3	3	3	
		3			

Figure 2.5

1	1	1	1	2	2
	1	3	3	3	
		3			

1	1	1	1	2	2
	1	3	3	3	
		3			

Figure 2.6

Figure 2.5 shows an example of a shifted rim hook tableau P of shape $(6,4,1)$ and content $(5,2,4)$. Here $r(\kappa\langle 1\rangle) = 1$, $r(\kappa\langle 2\rangle) = 0$ and $r(\kappa\langle 3\rangle) = 1$. Also $w(\kappa\langle 1\rangle) = -2$, $w(\kappa\langle 2\rangle) = 1$ and $w(\kappa\langle 3\rangle) = -1$. Hence $w(P) = (-2)\cdot(1)\cdot(-1) = 2$.

Suppose P is a shifted rim hook tableau. Then we denote by P_2 one of the tableaux obtained from P by circling or not circling the second tail of each double rim hook in P. The P_2 is called a *second tail circled rim hook tableau.* We use the notation $|\cdot|$ to refer to the uncircled version; e.g., $|P_2| = P$. See Figure 2.6 for examples of second tail circled rim hook tableaux.

We now define a new weight function w' for second tail circled rim hook tableaux. If τ is a rim hook of P_2, we define $w'(\tau) = (-1)^{r(\tau)}$. The weight $w'(P_2)$ is the product of the weights of rim hooks in P_2.

For each double rim hook τ of a rim hook tableau P, there are two second circled rim hooks τ_1, τ_2 such that $w(\tau) = w'(\tau_1) + w'(\tau_2)$. This fact implies the following:

PROPOSITION 2.1. *Let $\gamma \in OP$. Then we have*

$$\sum_{P} w(P) = \sum_{P_2} w'(P_2),$$

where the left-hand sum is over all shifted rim hook tableaux P of shape λ/μ and content γ, while the right-hand sum is over all shifted second tail circled rim hook tableaux P_2 of shape λ/μ and content γ.

3. Switching rule on the shifted rim hook tableaux

In [9] Stanton and White give the switching rule for rim hook tableau. It shows that the sum of the signs of rim hook tableaux of a given shape and content is independent of the order of the content. In this section we give a shifted rim hook analog of this switching procedure. Our switching algorithm shows that the sum of the weights of the shifted rim hook tableaux of a given shape and content does not depend on the order of the content if content parts are all odd. This gives us the combinatorial proof for the invariance of spin characters of $\tilde{S}_n$. See [8] for detail.

P_2 is said to be a *shifted second tail circled $*$-rim hook tableau* if P_2 is a shifted second tail circled rim hook tableau whose entries include $*$ and are from the set $\{1, 2, \ldots, m, *\}$, where $r - 1 < * < r$ for some integer r. We introduce the symbol $*$ to make it clear that no established order relationship governs $*$. We say that $*$ is covered by r (denoted by $* \lessdot r$) if r is the next integer larger than $*$ in P_2.

From now on, unless we explicitly specify to the contrary, we assume P_2 is a shifted second tail circled $*$-rim hook tableau of shape λ and contents all odd and $* \lessdot r$ in P_2. The circling of the second tail is necessary to compensate for the weight of 2 on double rim hooks.

If $\kappa\langle * \rangle \cup \kappa\langle r \rangle$ is disconnected in P_2, we call $*$ and r *disconnected*. We say that $*$ and r is a *single* (resp.,*double*) *rim hook union* if $\kappa\langle * \rangle \cup \kappa\langle r \rangle$ is a single (resp., double) rim hook. If $\kappa\langle * \rangle \cup \kappa\langle r \rangle$ is neither disconnected nor any rim hook union, we call $*$ and r *overlapping*.

We define an assignment $X(*)$ that sends P_2 into another shifted second tail circled $*$-rim hook tableau $\hat{P}_2$ of shape λ as follows:

1. If $*$ and r are disconnected in P_2, then $X(*)P_2 = \hat{P}_2 = P_2$, but with $r \lessdot *$.

2. If $*$ and r is a single rim hook union, then $X(*)$ moves all of the symbols at the head of $\tau = \kappa\langle * \rangle \cup \kappa\langle r \rangle$ to the tail of τ, and vice versa. The number of r's and $*$'s is preserved. In this case, either $r \lessdot *$ in $\hat{P}_2$ or $* \lessdot r$ in $\hat{P}_2$. Figure 3.1 gives us an example for case 2 with $* \lessdot r$ in $\hat{P}_2$ and Figure 3.2 shows case 2 with $r \lessdot *$ in $\hat{P}_2$.

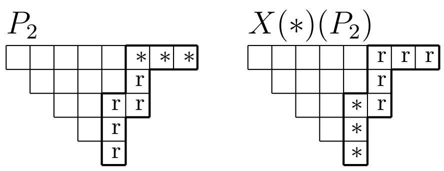

Figure 3.1

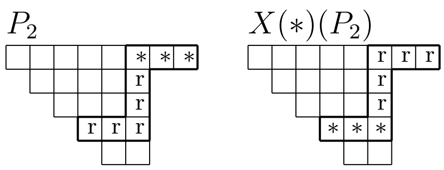

Figure 3.2

3. If $*$ and r is a double rim hook union, let $\tau = \kappa\langle * \rangle \cup \kappa\langle r \rangle$. Recall that we can write τ as follows: $\tau = \beta_1 \cup \beta_2 = \gamma_1 \cup \gamma_2 = \mathcal{A} \cup \alpha_1$.

Let $a = |\kappa\langle * \rangle|, b = |\kappa\langle r \rangle|$ and $c = |\beta_1| = |\gamma_1|$. Then we have

$$
\begin{aligned}
|\beta_2| &= |\gamma_2| = a + b - c, \\
|\alpha_1| &= 2c - a - b \quad \text{and} \\
|\mathcal{A}| &= 2(a + b - c).
\end{aligned}
$$

We say we *fill* τ from β_1 if the word with a $*$'s followed by b r's is inserted in τ, starting at the head of β_1, running down β_1 to the diagonal, then up β_2. Similarly, define filling τ from β_2, from γ_1 and from γ_2.

It is not hard to verify the following two lemmas. For examples, see Figure 3.3 and Figure 3.4.

Lemma 3.1. *If $a, b \neq |\mathcal{A}|/2$ then there are exactly two shifted skew rim hook tableaux of shape τ with a $*$'s and b r's. One of these (say T_1) fills τ from β_1 or from γ_1. The other (say T_2) fills τ from β_2 or from γ_2. If $* < r$ in T_1 and T_2 or if $r < *$ in T_1 and T_2, then $w(T_1) = -w(T_2)$. Otherwise, $w(T_1) = w(T_2)$.*

Lemma 3.2. *If $a = |\mathcal{A}|/2$ (resp., $b = |\mathcal{A}|/2$), then there are exactly three shifted skew rim hook tableaux of shape τ with a $*$'s and b r's. In one of these (say T_4), β_2 will contain the $*$'s (resp., r's). In the second (say T_5), γ_2 will contain the $*$'s (resp., r's). The third (say T_6) fills τ from β_1 or from γ_1 (resp., from β_2 or from γ_2). Also, $w(T_4) = -w(T_5)$ and if $* < r$ in T_6 then $w(T_6) = w(T_4) - w(T_5)$ (resp., $w(T_6) = w(T_5) - w(T_4)$) while if $r < *$ in T_6 then $w(T_6) = w(T_5) - w(T_4)$ (resp., $w(T_6) = w(T_4) - w(T_5)$).*

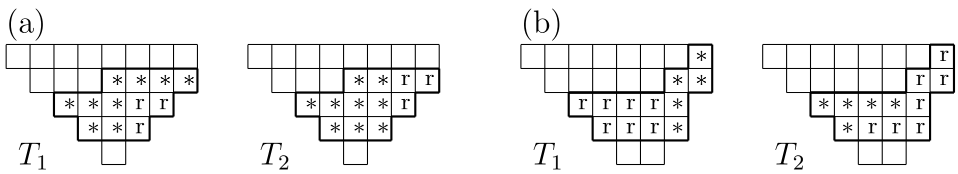

Figure 3.3

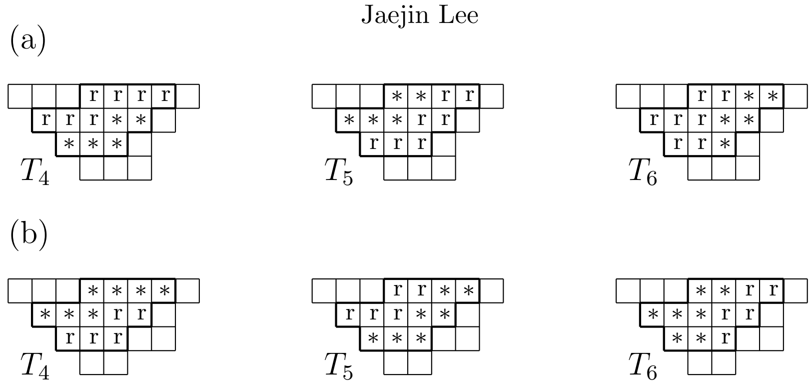

Figure 3.4

We now describe an assignment $X(*)P_2$ when $*$ and r is a double rim hook union in P_2. Suppose first $a, b \neq |\mathcal{A}|/2$. If P_2 contains T_1, then $X(*)P_2 = \hat{P}_2$ contains T_2, and vice versa. See Figure 3.5.

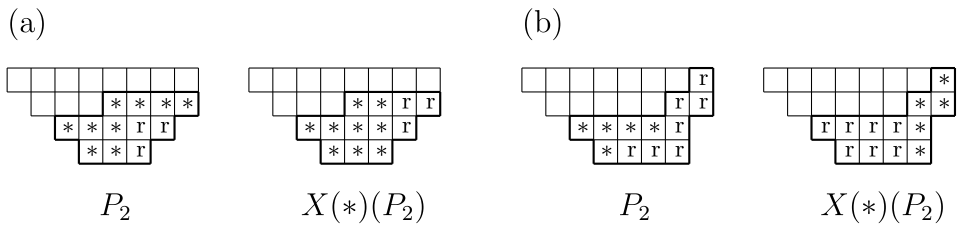

Figure 3.5

Suppose now $a = |\mathcal{A}|/2$ or $b = |\mathcal{A}|/2$. Say $a = |\mathcal{A}|/2$. Since $b = c$ and $* \lessdot r$ in P_2, P_2 cannot contain T_4. If P_2 contains T_5, then $\hat{P}_2$ contains T_6 with no circle on the second tail of τ; if P_2 contains T_6 with no circle on the second tail of τ, then $\hat{P}_2$ contains T_5; if P_2 contains T_6 with a circle on the second tail of τ, then $\hat{P}_2$ contains T_4. See Figure 3.6.

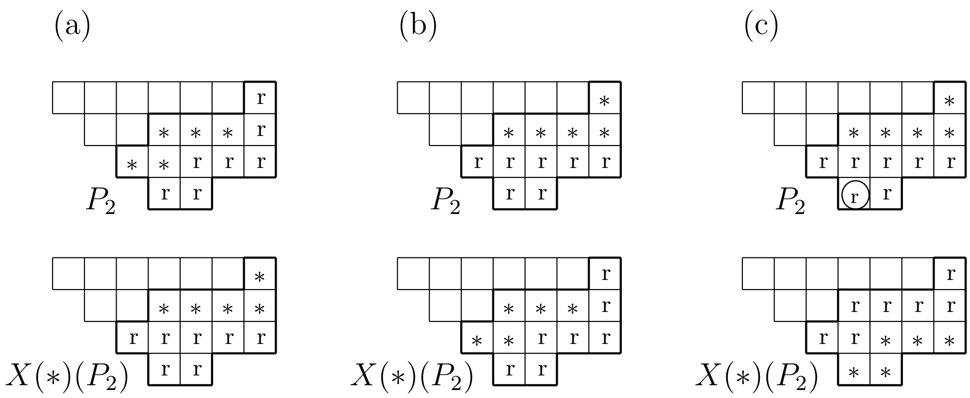

Figure 3.6

4. If $*$ and r is overlapping, then $X(*)$ exchanges $*$ and r along diagonals of P_2. See Figure 3.7.

P_2 $\qquad\qquad$ $X(*)(P_2)$

Figure 3.7

Proposition 3.3. *We have* $* \lessdot r$ *in* $\hat{P}_2 = X(*)P_2$ *if and only if*

$$w'(\kappa_{P_2}\langle * \rangle)w'(\kappa_{P_2}\langle r \rangle) = -w'(\kappa_{\hat{P}_2}\langle * \rangle)w'(\kappa_{\hat{P}_2}\langle r \rangle),$$

and $r \lessdot *$ *in* $\hat{P}_2 = X(*)P_2$ *if and only if*

$$w'(\kappa_{P_2}\langle * \rangle)w'(\kappa_{P_2}\langle r \rangle) = w'(\kappa_{\hat{P}_2}\langle * \rangle)w'(\kappa_{\hat{P}_2}\langle r \rangle).$$

Proof. It is easy to verify the above statements with a case-by-case argument. □

From Proposition 3.3 we have the following theorem:

Theorem 3.4. *Let* λ *be a partition with all distinct parts and* $\rho \in OP_n$ *and* ρ' *be any reordering of* ρ. *Then*

$$\sum_{P_2} w'(P_2) = \sum_{P_2'} w'(P_2'),$$

where the left-hand sum is over all shifted second circled rim hook tableaux P_2 *of shape* λ *and content* ρ, *and the right-hand sum is over all shifted second circled rim hook tableaux* P_2' *of shape* λ *and content* ρ'.

Proof. If ρ and ρ' differ by an adjacent transposition, X defined above establishes this identity. The theorem follows because any reordering can be written as a sequence of adjacent transpositions. The "signed bijection" in the general case is given by the involution principle of Garsia and Milne [2]. See [9] for details. □

Theorem 3.4 and Proposition 2.1 imply the following corollaries:

Corollary 3.5. *Let $\lambda \in DP_n$. Let ρ have all odd parts and ρ' be any reordering of ρ. Then*

$$\sum w(P) = \sum w(P'),$$

where the left-hand sum is over all shifted rim hook tableaux P of shape λ and content ρ, and the right-hand sum is over all shifted rim hook tableaux P' of shape λ and content ρ'.

4. The jeu de taquin on the shifted rim hook tableau

Related in several ways to the Schensted correspondence is the "jeu de tarquin" of Schützenberger[10]. It is an algorithm defined on column strict tableaux. An essentially similar procedure of switching values is described by Bender and Knuth[1]. In this section we will describe the jeu de taquin on the shifted rim hook tableau using the shifted rim hook switching procedure described in the previous section.

We need some assumption and notation in order to define the shifted rim hook jeu de tarquin from the operator X defined in Section 3. Let k be a fixed odd positive integer. We now assume that all shifted rim hooks are of length k and all shifted rim hook tableaux(and shifted $*$-rim hook tableaux) are shifted k-rim hook tableaux. If P_2 is a shifted second tail circled rim hook tableau containing a shifted rim hook of s's, then $Change(s,*)(P_2)$ is the shifted second tail circled $*$-rim hook tableau obtained by replacing every s in P_2 with $*$. The obvious convention $s-1 < * < s+1$ is also assumed. If P_2 is a shifted second tail circled rim hook tableau whose largest value is m, then $Erase(m)(P_2)$ is the shifted second tail circled rim hook tableau with the m's erased. Note that P_2 can be a shited second tail circled $*$-rim hook tableau with $*$ being the largest value. Then $\mathrm{Erase}(*)(P_2)$ erases the $*$'s in P_2.

Finally we come to the shifted rim hook analog of the Schützenberger evacuation procedure. Suppose P_2 is a shifted second tail circled rim hook tableau which contains a shifted rim hook of s's and there are ℓ values in P_2 larger than s. Let $E_s(P_2) = \mathrm{Erase}(*)\circ X^{\ell}(*)\circ \mathrm{Change}(s,*)(P_2)$. The operator E_s essentially "evacuates" s from P_2 by "erasing" s and successively "sliding" larger valued shifted rim hooks into the vacated positions. When the vacated positions reach the outer rim, the sliding stops.

Figure 4.1 gives an example of E_s.

$$P_2 = \begin{array}{ccccccccc} 1 & 1 & 1 & 1 & 2 & 2 & 4 & 4 & 4 \\ & 1 & 2 & 2 & 2 & 3 & 4 & & \\ & & 3 & 3 & 3 & 3 & 4 & & \\ & & & 5 & 5 & 5 & 5 & & \\ & & & & 5 & & & & \end{array} \qquad E_1(P_2) = \begin{array}{ccccccccc} 2 & 2 & 2 & 2 & 3 & 4 & 4 & 4 & 4 \\ & 2 & 3 & 3 & 3 & 4 & 5 & & \\ & & 3 & 5 & 5 & 5 & 5 & & \end{array}$$

Figure 4.1

We conclude this section by describing the shifted rim hook analog of the Schützenberger "evacuation tableau." Suppose P_2 is a shifted second tail circled rim hook tableau with entries $1 \leq s_1 < s_2 < \cdots < s_\ell \leq m$. Since $E_s(P_2)$ differs in shape from P_2 by a shifted rim hook outside the shape of $E_s(P_2)$, we can construct a shifted second tail circled rim hook tableau by successively evacuating $s_1, s_2, \cdots, s_\ell$ from P_2; and, after each evacuation, inserting $m+1-s_1, m+1-s_2, \cdots, m+1-s_\ell$ respectively into these shited rim hooks. This new tableau has the same shape as P_2 and is called $\sum(P_2)$. If $\ell = m$ so that the entries of P_2 are $\{1, 2, \ldots, m\}$, then the entries of $\sum(P_2)$ are also $\{1, 2, \ldots, m\}$.

More precisely, we can define $\sum(P_2)$ inductively as follows. Suppose m is given and P_2 is a shited second tail circled rim hook tableau whose content is $\{s_1, s_2, \ldots, s_\ell\}$ with $1 \leq s_1 < s_2 < \cdots < s_\ell \leq m$. We define $\sum(P_2)$ as the shifted rim hook tableau with the same shape as P_2, and with content $\{m+1-s_\ell, \ldots, m+1-s_2, m+1-s_1\}$, such that

$$E_{m+1-s_1} \circ \sum(P_2) = \sum(E_{s_1}(P_2)).$$

Note that since $m+1-s_1$ is the largest entry of $\sum(P_2)$, in this case $E_{m+1-s_1} = \text{Erase}(m+1-s_1)$. In Figure 4.2 we give an example of a pair P_2 and $\sum(P_2)$.

$$P_2 = \begin{array}{ccccccccc} 1 & 1 & 1 & 2 & 2 & 4 & 4 & 4 & 4 \\ & 1 & 1 & 2 & 3 & 4 & 5 & & \\ & & 2 & 2 & 3 & 5 & 5 & & \\ & & & 3 & 3 & 5 & & & \\ & & & & 3 & 5 & & & \end{array} \qquad \sum(P_2) = \begin{array}{ccccccccc} 1 & 1 & 1 & 1 & 1 & 4 & 5 & 5 & 5 \\ & 2 & 2 & 2 & 3 & 4 & 5 & & \\ & & 2 & 2 & 3 & 4 & 5 & & \\ & & & 3 & 3 & 4 & & & \\ & & & & 3 & 4 & & & \end{array}$$

Figure 4.2

In subsequent paper we will show that all connections between the Schensted correspondence for shifted tableaux and shifted jeu de tarquin have shifted rim hook versions.

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