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DUOTRIGINTIC FUNCTIONAL EQUATION AND ITS STABILITY IN BANACH SPACES

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Abstract. In this paper, we introduce a duotrigintic functional equation. Furthermore, we study the Hyers-Ulam stability of a duotrigintic functional equation in Banach spaces by using the direct method.

1. Introduction and preliminaries

Stability problem of a functional equation was first posed by Ulam [32] and that was partially answered by Hyers [7] and then generalized by Aoki [1] and Rassias [29] for additive mappings and linear mappings, respectively. In 1994, a generalization of Rassias theorem was obtained by Gâvruta [6], who replaced $\epsilon\left(\|x\|^p + \|y\|^p\right)$ by a general control function $\phi(x, y)$. After that, the general stability problems of various functional equations such as additive [9–11], quadratic [17], cubic [4, 8, 18], quartic [3, 19, 25], quintic [20, 33], sextic [33], nonic [5, 22, 23], decic [2], undecic [30], quattuordecic [31], hexadecic [24], octadecic [12], vigintic [16], viginticduo [21], quattuorvigintic [13, 27, 28], octavigintic [26] and trigintic [14] functional equations have been investigated by a number of authors with more general domains and co-domains.

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Let X and Y be real vector spaces. For convenience, we use the following abbreviation for a mapping $f : X \to Y$

$Df(x,y) = f(x+16y) - 32f(x+15y) + 496f(x+14y) - 4960f(x+13y)$
$+35960f(x+12y) - 201376f(x+11y) + 906192f(x+10y) - 3365856f(x+9y)$
$+10518300f(x+8y) - 28048800f(x+7y) + 64512240f(x+6y)$
$-129024480f(x+5y) + 225792840f(x+4y) - 347373600f(x+3y)$
$+471435600f(x+2y) - 565722720f(x+y) + 601080390f(x)$
$-565722720f(x-y) + 471435600f(x-2y) - 347373600f(x-3y)$
$+225792840f(x-4y) - 129024480f(x-5y) + 64512240f(x-6y)$
$-28048800f(x-7y) + 10518300f(x-8y) - 3365856f(x-9y)$
$+906192f(x-10y) - 201376f(x-11y) + 35960f(x-12y) - 4960f(x-13y) +$
$496f(x-14y) - 32f(x-15y) + f(x-16y) - 32!f(y)$ for all $x, y \in X$, where $32! = 2.631308369 \times 10^{35}$.

In this paper, we introduce the duotrigintic functional equation

$$Df(x,y) = 0 \tag{1.1}$$

for all $x, y \in X$. Moreover, we prove the Hyers-Ulam stability of the duotrigintic functional equation (1.1) in Banach spaces. Since $f(x) = x^{32}$ is a solution of (1.1), we say that the functional equation is a duotrigintic functional equation. Every solution of the duotrigintic equation is said to be a duotrigintic mapping.

THEOREM 1.1. *[15] Let X and Y be vector spaces. If $f : X \to B$ is the function (1.1) for all $x, y \in X$, then f is a duotrigintic mapping.*

2. Hyers-Ulam stability of the duotrigintic functional equation (1.1)

THEOREM 2.1. *Let $j = \pm 1$ and $\alpha : X^2 \to [0, \infty)$ be a function such that $\sum_{i=0}^{\infty} \frac{\alpha(2^{ij}x, 2^{ij}y)}{2^{32ij}}$ converges for all $x, y \in X$. Let $f : X \to Y$ be a mapping satisfying the inequality*

$$\|Df(x,y)\| \leq \alpha(x,y) \tag{2.2}$$

for all $x, y \in X$. Then there exists a unique mapping $G : X \to Y$ which satisfies (1.1) and

$$\|f(x) - G(x)\| \leq \frac{1}{2^{32}} \sum_{i=\frac{1-j}{2}}^{\infty} \frac{\alpha(2^{ij}x, 2^{ij}x)}{2^{32ij}} \tag{2.3}$$

for all $x \in X$.

Proof. Letting $x = 0$ and $y = 0$ in (1.1), we obtain that $f(0) = 0$. Replacing (x, y) by (x, x) and $(x, -x)$ in (1.1), respectively, and subtracting two resulting equations, we obtain $f(-x) = f(x)$, that is to say, f is an even mapping.
Replacing (x, y) by $(16x, x)$ and $(0, 2x)$, respectively, in (1.1), and subtracting the two resulting equations, we obtain

$$\begin{aligned}
&\|32f(31x) - 528f(30x) + 4960f(29x) - 35464f(28x) + 201376f(27x)\\
&-911152f(26x) + 3365856f(25x) - 10482340f(24x) + 28048800f(23x)\\
&-64713616f(22x) + 129024480f(21x) - 224886648f(20x) + 347373600f(19x)\\
&-474801456f(18x) + 565722720f(17x) - 590562090f(16x) + 565722720f(15x)\\
&-499484400f(14x) + 347373600f(13x) - 161280600f(12x) + 129024480f(11x)\\
&-193536720f(10x) + 28048800f(9x) + 215274540f(8x) + 3365856f(7x)\\
&-348279792f(6x) + 201376f(5x) + 471399640f(4x) + 4960f(3x)\\
&-\frac{32!}{2}f(2x) + 32!f(x)\Big\| \leq \alpha(0, 2x) + \alpha(16x, x) \qquad (2.4)
\end{aligned}$$

for all $x \in X$. Substituting $x = 15x$ and $y = x$ in (1.1), further multiplying the resulting equation by 32, and subtracting the obtained result from (2.4), we get

$$\begin{aligned}
&\|496f(30x) - 10912f(29x) + 123256f(28x) - 949344f(27x) + 5532880f(26x)\\
&-25632288f(25x) + 97225052f(24x) - 308536800f(23x) + 852847984f(22x)\\
&-1935367200f(21x) + 3903896712f(20x) - 6877997280f(19x) + 1.064115374 \times 10^{10}f(18x)\\
&-1.452021648 \times 10^{10}f(17x) + 1.751256495 \times 10^{10}f(16x) - 1.866884976 \times 10^{10}f(15x)\\
&+1.760364264 \times 10^{1}0f(14x) - 1.47385656 \times 10^{10}f(13x) + 1.09546746 \times 10^{10}f(12x)\\
&-7096346400f(11x) + 3935246640f(10x) - 2036342880f(9x) + 1112836146f(8x)\\
&-333219744f(7x) - 240572400f(6x) - 28796768f(5x) + 477843672f(4x) - 1145760f(3x)\\
&-\frac{32!}{2}f(2x) + 30!(33)f(x)\Big\| \leq \alpha(0, 2x) + \alpha(16x, x) + 32\alpha(15x, x) \qquad (2.5)
\end{aligned}$$

for all $x \in X$. Letting $x = 14x$ and $y = x$ in (1.1), further multiplying the resulting equation by 496, and subtracting the obtained result from (2.5), we arrive

$$\begin{aligned}
&\|4960f(29x) - 122760f(28x) + 1510816(27x) - 12303280f(26x) + 74250208f(25x)\\
&-352246180f(24x) + 1360927776f(23x) - 43842288f(22x) + 1.19768376 \times 10^{10}f(21x)\\
&-2.809417433 \times 10^{10}f(20x) + 5.71181448 \times 10^{10}f(19x) - 1.013520949 \times 10^{11}f(18x)\\
&+1.577770891 \times 10^{11}f(17x) - 2.163194927 \times 10^{11}f(16x) + 2.619296193 \times 10^{11}f(15x)\\
&-2.805322308 \times 10^{11}f(14x) + 2.658599035 \times 10^{11}f(13x) - 2.22877383 \times 10^{11}f(12x)\\
&+1.652009592 \times 10^{11}f(11x) - 1.08058002 \times 10^{11}f(10x) + 6.19597992 \times 10^{1}0f(9x)\\
&-3.08852349 \times 10^{10}f(8x) + 1.357898506 \times 10^{10}f(7x) - 5457649200f(6x)\\
&+1640667808f(5x) + 28372440f(4x) + 98736736f(3x) - \frac{32!}{2}f(2x) + 32!(529)f(x)\Big\|\\
&\leq [\alpha(0, 2x) + \alpha(16x, x) + 32\alpha(15x, x) + 496\alpha(14x, x)] \qquad (2.6)
\end{aligned}$$

for all $x \in X$. Replacing x by $13x$ and y by x in (1.1), further multiplying the resulting equation by 4960, and subtracting the obtained result from (2.6), we get

$$\begin{aligned}
&\|35960f(28x) - 949344f(27x) + 12298320f(26x) - 104111392f(25x) + 646578780f(24x)\\
&-3133784544f(23x) + 1.231041694 \times 10^{10} f(22x) - 4.01939304 \times 10^{10} f(21x)\\
&+1.110278737 \times 10^{11} f(20x) - 2.628625656 \times 10^{11} f(19x) + 5.386093259 \times 10^{11} f(18x)\\
&-9.621553973 \times 10^{11} f(17x) + 1.506653563 \times 10^{12} f(16x) - 2.076390957 \times 10^{12} f(15x)\\
&+2.52545246 \times 10^{12} f(14x) - 2.715498831 \times 10^{12} f(13x) + 2.583107308 \times 10^{12} f(12x)\\
&-2.173119617 \times 10^{12} f(11x) + 1.614915054 \times 10^{12} f(10x) - 1.057972687 \times 10^{12} f(9x)\\
&+6.090761859 \times 10^{11} f(8x) - 3.064017253 \times 10^{11} f(7x) + 1.336643988 \times 10^{11} f(6x)\\
&-5.053010019 \times 10^{10} f(5x) + 1.67230182 \times 10^{10} f(4x) - 4395980544f(3x)\\
&-\frac{32!}{2} f(2x) + 32!(5489)f(x)\bigg\| \leq [\alpha(0, 2x) + \alpha(16x, x)\\
&\qquad +32\alpha(15x, x) + 496\alpha(14x, x) + 4960\alpha(13x, x)] \qquad (2.7)
\end{aligned}$$

for all $x \in X$. Taking $x = 12x$ and replacing $y = x$ in (1.1), further multiplying the resulting equation by 35960, and subtracting the obtained result from (2.7), we have

$$\begin{aligned}
&\|201376f(27x) - 5537840f(26x) + 74250208f(25x) - 646542820f(24x) + 4107696416f(23x)\\
&-2.027624738 \times 10^{10} f(22x) + 8.084225136 \times 10^{10} f(21x) - 2.672101943 \times 10^{11} f(20x)\\
&+7.457722824 \times 10^{11} f(19x) - 1.781250825 \times 10^{12} f(18x) + 3.677564904 \times 10^{12} f(17x)\\
&-6.612856963 \times 10^{12} f(16x) + 1.04151637 \times 10^{13} f(15x) - 1.442737172 \times 10^{13} f(14x)\\
&+1.762789018 \times 10^{13} f(13x) - 1.903174351 \times 10^{13} f(12x) + 1.817026939 \times 10^{13} f(11x)\\
&-1.533790913 \times 10^{13} f(10x) + 1.43358197 \times 10^{13} f(9x) - 7.51043434 \times 10^{12} f(8x)\\
&+4.333318575 \times 10^{12} f(7x) - 2.186195751 \times 10^{12} f(6x) + 9.581047478 \times 10^{11} f(5x)\\
&-3.615150858 \times 10^{11} f(4x) + 1.16641352 \times 10^{11} f(3x) - \tfrac{32!}{2} f(2x) + 32!(41449)f(x)\|\\
&\qquad \leq [\alpha(0, 2x) + \alpha(16x, x) + 32\alpha(15x, x)\\
&\qquad\quad +496\alpha(14x, x) + 4960\alpha(13x, x) + 35960\alpha(12x, x)] \qquad (2.8)
\end{aligned}$$

for all $x \in X$. Taking $x = 11x$ and replacing $y = x$ in (1.1), further multiplying the resulting equation by 201376, and subtracting the obtained result from (2.8), we get

$$\begin{aligned}
&\|906192f(26x) - 25632288f(25x) + 352282140f(24x) - 3133784544f(23x)\\
&+2.0276046 \times 10^{10} f(22x) - 1.016430688 \times 10^{11} f(21x) + 4.105924235 \times 10^{11} f(20x)\\
&-1.372360898 \times 10^{12} f(19x) + 3.867104325 \times 10^{12} f(18x) - 9.313651939 \times 10^{12} f(17x)\\
&+1.936957672 \times 10^{13} f(16x) - 3.505409525 \times 10^{13} f(15x) + 5.552533436 \times 10^{13} f(14x)\\
&-7.730792521 \times 10^{13} f(13x) + 9.489123499 \times 10^{13} f(12x) - 1.028728952 \times 10^{14} f(11x)\\
&+9.858506937 \times 10^{13} f(10x) - 8.350223342 \times 10^{13} f(9x) + 6.244227173 \times 10^{13} f(8x)\\
&-4.113594037 \times 10^{13} f(7x) + 2.379623793 \times 10^{13} f(6x) - 1.203311299 \times 10^{13} f(5x)\\
&+5.286846507 \times 10^{12} f(4x) - 2.001591711 \times 10^{12} f(3x) - \frac{32!}{2} f(2x) + 32!(242825)f(x)\bigg\|\\
&\qquad \leq [\alpha(0, 2x) + \alpha(16x, x) + 32\alpha(15x, x) + 496\alpha(14x, x) + 4960\alpha(13x, x)\\
&\qquad\quad +35960\alpha(12x, x) + 201376\alpha(11x, x)] \qquad (2.9)
\end{aligned}$$

for all $x \in X$. Letting $x = 10x$ and $y = x$ in (1.1), further multiplying the resulting equation by 906192, and subtracting the obtained result from (2.9), we have

$$\begin{aligned}&\|3365856f(25x) - 97189092f(24x) + 1360927776f(23x) - 1.231061832 \times 10^{10}f(22x)\\ &+8.084225136 \times 10^{10}f(21x) - 4.105915173 \times 10^{11}f(20x) + 1.677750882 \times 10^{12}f(19x)\\ &-5.664494989 \times 10^{12}f(18x) + 1.610394623 \times 10^{13}f(17x) - 3.909089907 \times 10^{13}f(16x)\\ &+8.186685635 \times 10^{13}f(15x) - 1.490863309 \times 10^{14}f(14x) + 2.374792521 \times 10^{14}f(13x)\\ &-3.323199342 \times 10^{14}f(12x) + 4.097805079 \times 10^{14}f(11x) - 4.461091714 \times 10^{14}f(10x)\\ &+4.291511697 \times 10^{14}f(9x) - 3.647688975 \times 10^{14}f(8x) + 2.73651237 \times 10^{14}f(7x)\\ &-1.808154283 \times 10^{14}f(6x) + 1.048878683 \times 10^{14}f(5x) - 5.317407875 \times 10^{13}f(4x)\\ &+2.342050117 \times 10^{13}f(3x) - \tfrac{32!}{2}f(2x) + 32!(1149017)f(x)\|\\ &\quad \le [\alpha(0,2x) + \alpha(16x,x) + 32\alpha(15x,x) + 496\alpha(14x,x) + 4960\alpha(13x,x)\\ &\qquad +35960\alpha(12x,x) + 201376\alpha(11x,x) + 906192\alpha(10x,x)]\end{aligned} \tag{2.10}$$

for all $x \in X$. Setting $x = 9x$ and $y = x$ in (1.1), further multiplying the resulting equation by 3365856, and subtracting the obtained result from (2.10), we get

$$\begin{aligned}&\|10518300f(24x) - 308536800f(23x) + 4384027440f(22x) - 4.01939304 \times 10^{10}f(21x)\\ &+2.672111005 \times 10^{11}f(20x) - 1.372360898 \times 10^{12}f(19x) + 5.664491624 \times 10^{12}f(18x)\\ &-1.929913693 \times 10^{13}f(17x) + 5.53173227 \times 10^{13}f(16x) - 1.352720538 \times 10^{14}f(15x)\\ &+2.851914892 \times 10^{14}f(14x) - 5.225069332 \times 10^{14}f(13x) + 8.368895818 \times 10^{14}f(12x)\\ &-1.177003885 \times 10^{15}f(11x) + 1.45803204 \times 10^{15}f(10x) - 1.593998867 \times 10^{15}f(9x)\\ &+1.539372314 \times 10^{15}f(8x) - 1.313133109 \times 10^{15}f(7x) + 9.883941957 \times 10^{14}f(6x)\\ &-6.550999864 \times 10^{14}f(5x) + 3.811204361 \times 10^{14}f(4x) - 1.938394451 \times 10^{14}f(3x)\\ &-\tfrac{32!}{2}f(2x) + 32!(4514873)f(x)\| \le [\alpha(0,2x) + \alpha(16x,x) + 32\alpha(15x,x)\\ &+496\alpha(14x,x) + 4960\alpha(13x,x) + 35960\alpha(12x,x) + 201376\alpha(11x,x)\\ &\qquad +906192\alpha(10x,x) + 3365856\alpha(9x,x)]\end{aligned} \tag{2.11}$$

for all $x \in X$. Taking $x = 8x$ and replacing $y = x$ in (1.1), further multiplying the resulting equation by 10518300, and subtracting the obtained result from (2.11), we get

$$\begin{aligned}&\|28048800f(23x) - 833049360f(22x)1.19768376 \times 10^{10}f(21x) - 1.110269675 \times 10^{11}f(20x)\\ &+7.457722824 \times 10^{11}f(19x) - 3.867107693 \times 10^{12}f(18x) + 1.610394623 \times 10^{13}f(17x)\\ &-5.53173122 \times 10^{13}f(16x) + 1.597536392 \times 10^{14}f(15x) - 3.933676047 \times 10^{14}f(14x)\\ &+8.346112548 \times 10^{14}f(13x) - 1.538067247 \times 10^{15}f(12x) + 2.476775902 \times 10^{15}f(11x)\\ &-3.500669031 \times 10^{15}f(10x) + 4.356442419 \times 10^{15}f(9x) - 4.782971563 \times 10^{15}f(8x)\\ &+4.637308513 \times 10^{15}f(7x) - 3.970312093 \times 10^{15}f(6x) + 2.998731922 \times 10^{15}f(5x)\\ &-1.994214631 \times 10^{15}f(4x) + 1.165396876 \times 10^{15}f(3x) - \tfrac{32!}{2}f(2x) + 32!(15033173)f(x)\|\\ &\le [\alpha(0,2x) + \alpha(16x,x) + 32\alpha(15x,x) + 496\alpha(14x,x) + 4960\alpha(13x,x)\\ &\qquad +201376\alpha(11x,x) + 906192\alpha(10x,x) + 3365856\alpha(9x,x) + 10518300\alpha(8x,x)]\end{aligned} \tag{2.12}$$

for all $x \in X$. Letting $x = 7x$ and $y = x$ in (1.1),further multiplying the resulting equation 28048800, and subtracting the obtained result from

(2.12), we obtain

$$\begin{aligned}
&\|64512240f(22x) - 1935367200f(21x)\\
&+2.809508052 \times 10^{10}f(20x) - 2.6286625656 \times 10^{11}f(19x)\\
&+1.781247459 \times 10^{12}f(18x) - 9.313651939 \times 10^{12}f(17x) + 3.909090958 \times 10^{13}f(16x)\\
&-1.352720538 \times 10^{14}f(15x) + 3.933675767 \times 10^{14}f(14x) - 9.748796622 \times 10^{14}f(13x)\\
&+2.080914588 \times 10^{15}f(12x) - 3.856442309 \times 10^{15}f(11x) + 6.242743601 \times 10^{15}f(10x)\\
&-8.866760471 \times 10^{15}f(9x) + 1.108487277 \times 10^{16}f(8x) - 1.222228904 \times 10^{16}f(7x)\\
&+1.189767046 \times 10^{16}f(6x) - 1.022547957 \times 10^{16}f(5x) + 7.754846356 \times 10^{15}f(4x)\\
&-5.193238933 \times 10^{15}f(3x) - \frac{32!}{2}f(2x) + 32!(43081973)f(x)\Big\|
\end{aligned}$$

$$\begin{aligned}
&\leq [\alpha(0, 2x) + \alpha(16x, x) + 32\alpha(15x, x) + 496\alpha(14x, x) + 4960\alpha(13x, x)\\
&\quad +35960\alpha(12x, x) + 201376\alpha(11x, x) + 906192\alpha(10x, x) + 3365856\alpha(9x, x)\\
&\quad +10518300\alpha(8x, x) + 28048800\alpha(7x, x)] \qquad (2.13)
\end{aligned}$$

for all $x \in X$. Setting $x = 6x$ and $y = x$ in (1.1), further multiplying the resulting equation by 64512240, and subtracting the obtained result from (2.13), we get

$$\begin{aligned}
&\Big\|129024480f(21x) - 3902990520f(20x) + 5.71181448 \times 10^{10}f(19x)\\
&-5.386126918 \times 10^{11}f(18x) + 3.677564904 \times 10^{12}f(17x) - 1.936956622 \times 10^{13}f(16x)\\
&+8.186685633 \times 10^{13}f(15x) - 2.851915174 \times 10^{14}f(14x) + 8.346112548 \times 10^{14}f(13x)\\
&-2.080914522 \times 10^{15}f(12x) + 4.467215911 \times 10^{15}f(11x) - 8.323658349 \times 10^{15}f(10x)\\
&+1.354309065 \times 10^{16}f(9x) - 1.93285258 \times 10^{16}f(8x) + 2.427407082 \times 10^{16}f(7x)\\
&-2.688169178 \times 10^{16}f(6x) + 2.628355153 \times 10^{16}f(5x) - 2.271698069 \times 10^{16}f(4x)\\
&+1.743374903 \times 10^{16}f(3x) - \tfrac{32!}{2}f(2x) + 32!(107594213)f(x)\Big\|
\end{aligned}$$

$$\begin{aligned}
&\leq [\alpha(0, 2x) + \alpha(16x, x) + 32\alpha(15x, x) + 496\alpha(14x, x) + 4960\alpha(13x, x)\\
&+35960\alpha(12x, x) + 201376\alpha(11x, x) + 906192\alpha(10x, x) + 3365856\alpha(9x, x)\\
&\quad +10518300\alpha(8x, x) + 28048800\alpha(7x, x) + 64512240\alpha(6x, x)] \qquad (2.14)
\end{aligned}$$

for all $x \in X$. Replacing $x = 5x$ and $y = x$ in (1.1), further multiplying the resulting equation by 129024480, and subtracting the obtained result from (2.14), we have

$$\begin{aligned}
&\Big\|225792840f(20x) - 6877997280f(19x)1.013487293 \times 10^{11}f(18x)\\
&-9.621553973 \times 10^{11}f(17x) + 6.612867481 \times 10^{12}f(16x) - 3.505409525 \times 10^{13}f(15x)\\
&+1.490863029 \times 10^{14}f(14x) - 5.225069332 \times 10^{14}f(13x) + 1.538067313 \times 10^{15}f(12x)\\
&-3.856442438 \times 10^{15}f(11x) + 8.323662221 \times 10^{15}f(10x) - 1.558977711 \times 10^{16}f(9x)\\
&+2.549181227 \times 10^{16}f(8x) - 3.655730204 \times 10^{16}f(7x) + 4.613637043 \times 10^{16}f(6x)\\
&-5.138745418 \times 10^{16}f(5x) + 5.07093769 \times 10^{16}f(4x) - 4.47501023 \times 10^{16}f(3x)\\
&-\tfrac{32!}{2}f(2x)f(2x) + 32!(236618693)f(x)\Big\|
\end{aligned}$$

$$
\begin{aligned}
&\le [\alpha(0,2x)+\alpha(16x,x)+32\alpha(15x,x)\\
&\quad +496\alpha(14x,x)+4960\alpha(13x,x)+35960\alpha(12x,x)+201376\alpha(11x,x)\\
&\quad +906192\alpha(10x,x)+3365856\alpha(9x,x)+10518300\alpha(8x,x)\\
&\quad +28048800\alpha(7x,x)+64512240\alpha(6x,x)+129024480\alpha(5x,x)]
\end{aligned} \tag{2.15}
$$

for all $x \in X$. Letting $x = 4x$ and $y = x$ in (1.1), further multiplying the resulting equation by 225792840, and subtracting the obtained result from (2.15), we have

$$
\begin{aligned}
&\|347373600f(19x)-1.064451964\times10^{10}f(18x)1.577770891\times10^{11}f(17x)\\
&-1.506643045\times10^{12}f(16x)+1.041516365\times10^{13}f(15x)-5.552536241\times10^{13}f(14x)\\
&+2.374792521\times10^{14}f(13x)-8.36889742\times10^{14}f(12x)+2.476782998\times10^{15}f(11x)\\
&-6.242851659\times10^{15}f(10x)+1.354414659\times10^{16}f(9x)-2.549871384\times10^{16}f(8x)\\
&+4.19226389\times10^{16}f(7x)-6.051502427\times10^{16}f(6x)+7.710867162\times10^{16}f(5x)\\
&-8.73852283\times10^{16}f(4x)+8.93192555\times10^{16}f(3x)-\frac{32!}{2}f(2x)+32!(462411533)f(x)\Big\|
\end{aligned}
$$

$$
\begin{aligned}
&\le [\alpha(0,2x)+\alpha(16x,x)+32\alpha(15x,x)+496\alpha(14x,x)+4960\alpha(13x,x)+35960\alpha(12x,x)\\
&\quad +201376\alpha(11x,x)+906192\alpha(10x,x)+3365856\alpha(9x,x)+10518300\alpha(8x,x)\\
&\quad +28048800\alpha(7x,x)+64512240\alpha(6x,x)+129024480\alpha(5x,x)+225792840\alpha(4x,x)]
\end{aligned} \tag{2.16}
$$

for all $x \in X$. Setting $x = 3x$ and $y = x$ in (1.1), further multiplying the resulting equation by 347373600, and subtracting the obtained result from (2.16), we have

$$
\begin{aligned}
&\|471435600f(18x)-1.45202165\times10^{10}f(17x)+2.16330011\times10^{11}f(16x)\\
&-2.076390956\times10^{12}f(15x)+1.442734367\times10^{13}f(14x)-7.73082726\times10^{13}f(13x)\\
&+3.3233089\times10^{14}f(12x)-1.77169036\times10^{15}f(11x)+3.502283946\times10^{15}f(10x)\\
&-8.87819402\times10^{15}f(9x)+1.939093697\times10^{16}f(8x)-3.682661996\times10^{16}f(7x)\\
&+6.132260323\times10^{16}f(6x)-9.030938968\times10^{16}f(5x)+1.88753222\times10^{17}f(4x)\\
&-1.418900525\times10^{17}f(3x)-\tfrac{32!}{2}f(2x)+32!(809785133)f(x)\|
\end{aligned}
$$

$$
\begin{aligned}
&\le [\alpha(0,2x)+\alpha(16x,x)+32\alpha(15x,x)+496\alpha(14x,x)+4960\alpha(13x,x)\\
&\quad +35960\alpha(12x,x)+201376\alpha(11x,x)+906192\alpha(10x,x)+3365856\alpha(9x,x)\\
&\quad +10518300\alpha(8x,x)+28048800\alpha(7x,x)+64512240\alpha(6x,x)+129024480\alpha(5x,x)\\
&\quad +225792840\alpha(4x,x)+347373600\alpha(3x,x)]
\end{aligned} \tag{2.17}
$$

for all $x \in X$. Letting $x = 2x$ and $y = x$ in (1.1), further multiplying the resulting equation by 471435600, and subtracting the obtained result from (2.17), we get

$$
\begin{aligned}
&\|565722700f(17x)-1.75020466\times10^{10}f(16x)+2.619296198\times10^{11}f(15x)\\
&-2.525951944\times10^{12}f(14x)+1.764262872\times10^{13}f(13x)-9.51141113\times10^{13}f(12x)\\
&+4.11953627\times10^{14}f(11x)-1.47336995\times10^{15}f(10x)+4.43994465\times10^{15}f(9x)
\end{aligned}
$$

$-1.144964077 \times 10^{16} f(8x) + 2.558689753 \times 10^{16} f(7x) - 5.008288087 \times 10^{16} f(6x)$
$+8.667539472 \times 10^{16} f(5x) - 1.337895693 \times 10^{17} f(4x) + 1.856385106 \times 10^{17} f(3x)$
$-\frac{32!}{2} f(2x) + 32!(1281220733) f(x)\|$

$$\begin{aligned}
&\le [\alpha(0,2x) + \alpha(16x,x) + 32\alpha(15x,x) \\
&\quad +496\alpha(14x,x) + 4960\alpha(13x,x) + 35960\alpha(12x,x) + 201376\alpha(11x,x) \\
&\quad +906192\alpha(10x,x) + 3365856\alpha(9x,x) + 10518300\alpha(8x,x) \\
&\quad +28048800\alpha(7x,x) + 64512240\alpha(6x,x) + 129024480\alpha(5x,x) \\
&\quad +225792840\alpha(4x,x) + 347373600\alpha(3x,x) + 471435600\alpha(2x,x)]
\end{aligned} \tag{2.18}$$

for all $x \in X$. Taking $x = x$ and replacing $y = x$ in (1.1), further multiplying the resulting equation by 565722720, and subtracting the obtained result from (2.18), we get

$\|601080390 f(16x) - 1.923457248 \times 10^{10} f(15x) + 2.981358734 \times 10^{11} f(14x)$
$-2.981358734 \times 10^{12} f(13x) + 2.161485082 \times 10^{13} f(12x) - 1.210431646 \times 10^{14} f(11x)$
$+5.446942408 \times 10^{14} f(10x) - 2.023150037 \times 10^{15} f(9x) + 6.32234387 \times 10^{15} f(8x)$
$-1.685958364 \times 10^{16} f(7x) + 3.877704238 \times 10^{16} f(6x) - 7.755678478 \times 10^{16} f(5x)$
$+1.357196483 \times 10^{17} f(4x) - 2.08799458 \times 10^{17} f(3x) - \frac{32!}{2} f(2x) + 32!(1846943453) f(x)\|$

$$\begin{aligned}
&\le [\alpha(0,2x) + \alpha(16x,x) + 32\alpha(15x,x) \\
&\quad +496\alpha(14x,x) + 4960\alpha(13x,x) + 35960\alpha(12x,x) + 201376\alpha(11x,x) \\
&\quad +906192\alpha(10x,x) + 3365856\alpha(9x,x) + 10518300\alpha(8x,x) \\
&\quad +28048800\alpha(7x,x) + 64512240\alpha(6x,x) + 129024480\alpha(5x,x) \\
&\quad +225792840\alpha(4x,x) + 347373600\alpha(3x,x) + 471435600\alpha(2x,x) + 565722720\alpha(x,x)]
\end{aligned} \tag{2.19}$$

for all $x \in X$. Taking $x = 0$ and replacing $y = x$ in (1.1), further multiplying the resulting equation by 601080390, and subtracting the obtained result from (2.19), we get

$$\begin{aligned}
&\left\| -\frac{32!}{2} f(2x) + 32!(2147483648) f(x) \right\| \\
&\le [\alpha(0,2x) + \alpha(16x,x) + 32\alpha(15x,x) \\
&\quad +496\alpha(14x,x) + 4960\alpha(13x,x) + 35960\alpha(12x,x) + 201376\alpha(11x,x) \\
&\quad +906192\alpha(10x,x) + 3365856\alpha(9x,x) + 10518300\alpha(8x,x) \\
&\quad +28048800\alpha(7x,x) + 64512240\alpha(6x,x) + 129024480\alpha(5x,x) \\
&\quad +225792840\alpha(4x,x) + 347373600\alpha(3x,x) + 471435600\alpha(2x,x) \\
&\quad +565722720\alpha(x,x) + 601080390\alpha(0,x)]
\end{aligned} \tag{2.20}$$

for all $x \in X$. Define

$$\begin{aligned}\alpha(x,x) = &\frac{1}{2.631308369 \times 10^{35}}\left[\alpha(0,2x) + \alpha(16x,x) + 32\alpha(15x,x)\right.\\ &+496\alpha(14x,x) + 4960\alpha(13x,x) + 35960\alpha(12x,x) + 201376\alpha(11x,x)\\ &+906192\alpha(10x,x) + 3365856\alpha(9x,x) + 10518300\alpha(8x,x)\\ &+28048800\alpha(7x,x) + 64512240\alpha(6x,x) + 129024480\alpha(5x,x)\\ &+225792840\alpha(4x,x) + 347373600\alpha(3x,x) + 471435600\alpha(2x,x)\\ &\left.+565722720\alpha(x,x) + 601080390\alpha(0,x)\right] \end{aligned} \tag{2.21}$$

for all $x \in X$. From (2.20), we get

$$\left\|f(2x) - 2^{32}f(x)\right\| \leq \alpha(x,x) \tag{2.22}$$

for all $x \in X$. It follows from (2.22) that

$$\left\|\frac{f(2x)}{2^{32}} - f(x)\right\| \leq \frac{\alpha(x,x)}{2^{32}} \tag{2.23}$$

for all $x \in X$. Now, replacing x by $2x$ and dividing 2^{32} in (2.23), we get

$$\left\|\frac{f(2^2x)}{2^{64}} - \frac{f(2x)}{2^{32}}\right\| \leq \frac{\alpha(2x,2x)}{2^{64}} \tag{2.24}$$

for all $x \in X$. From (2.23) and (2.24), we obtain

$$\left\|\frac{f(2^2x)}{2^{64}} - f(x)\right\| \leq \frac{1}{2^{32}}\left[\alpha(x,x) + \frac{\alpha(2x,2x)}{2^{32}}\right] \tag{2.25}$$

for all $x \in X$. So for any positive integer a, we get

$$\left\|\frac{f(2^ax)}{2^{a(32)}} - f(x)\right\| \leq \frac{1}{2^{32}}\sum_{i=0}^{a-1}\frac{\alpha(2^ix,2^ix)}{2^{i(32)}} \tag{2.26}$$

for all $x \in X$. To prove the convergence of sequence $\left\{\frac{f(2^ax)}{2^{a(32)}}\right\}$, replacing x by 2^ix in (2.26) and dividing the resultant by $2^{i(27)}$ for any $a, i > 0$, we get

$$\begin{aligned}\left\|\frac{f(2^{a+i}x)}{2^{a+i(32)}}-\frac{f(2^i x)}{2^{i(32)}}\right\| &= \frac{1}{2^{32i}}\left\|\frac{f(2^a.2^i x)}{2^{a(27)}}-f(2^i x)\right\| \\ &\le \frac{1}{2^{i(32)}}\frac{1}{2^{32}}\sum_{c=0}^{a-1}\frac{\alpha\left(2^c.2^i x, 2^c.2^i x\right)}{2^{c(32)}} \\ &\le \frac{1}{2^{32}}\sum_{c=0}^{\infty}\frac{\alpha\left(2^{c+i}x, 2^{c+i}x\right)}{2^{32(c+i)}} \\ &\to 0 \text{ as } i\to\infty\end{aligned}$$

for all $x \in X$. Thus, it follows that the sequence $\left\{\frac{f(2^a x)}{2^{a(32)}}\right\}$ is Cauchy in Y and so it converges. Therefore, the mapping $G : X \to Y$, defined by

$$G(x) = \lim_{a\to\infty}\frac{f(2^a x)}{2^{a(32)}},$$

is well defined for all $x \in X$. In order to show that G satisfies (1.1), by interchanging (x, y) by $(2^a x, 2^a y)$ in (2.2) and then dividing by $2^{a(27)}$, we get

$$\begin{aligned}\|G(x,y)\| &= \lim_{a\to\infty}\frac{1}{2^{a(32)}}\|Df_{32}(2^a x, 2^a y)\| \\ &\le \lim_{a\to\infty}\frac{1}{2^{a(32)}}\alpha(2^a x, 2^a y)\end{aligned}$$

for all $x \in X$. So the mapping G is duotrigintic.

Taking the limit as a approaches to infinity in (2.26), we find that the mapping G is a duotrigintic mapping satisfying the inequality (2.3). Hence G satisfies (1.1) for all $x, y \in X$. To prove that G is unique, we assume now that there is U as another duotrigintic mapping satisfying (1.1) and the inequality (2.3). Then

$$\begin{aligned}\|G(x)-U(x)\| &= \frac{1}{2^{a(32)}}\|G(2^a x)-U(2^a x)\| \\ &\le \frac{1}{2^{a(32)}}\left\{\|G(2^a x)-f(2^a x)\|+\|f(2^a x)-U(2^a x)\|\right\} \\ &\le \frac{1}{2^{32}}\sum_{i=0}^{\infty}\frac{\alpha(2^{c+a}x, 2^{c+a}x)}{2^{32(c+a)}}\end{aligned}$$

for all $x \in X$. Therefore, as $a \to \infty$ in the above inequality, we conclude that $G(x) = U(x)$, for all $x \in X$. Now, replacing x by $\frac{x}{2}$ in (2.22), we have

$$\left\| f(x) - 2^{32} f(\frac{x}{2}) \right\| \leq \alpha \left(\frac{x}{2}, \frac{x}{2} \right)$$

for all $x \in X$. The rest of the proof is similar to that of case $j = 1$. Thus, for $j = -1$ the assertion holds as well. This completes the proof. $\square$

3. Conclusion

In this paper, we have introduced a duotrigintic functional equation. Furthermore, we have studied the Hyers-Ulam stability of the duotrigintic functional equation in Banach spaces by using the direct method.

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